MEAN-FIELD HOMOGENIZATION OF ELASTO-(VISCO)PLASTIC COMPOSITES: FORMULATIONS FOR TIME-DEPENDENT AND INDEPENDENT BEHAVIORS

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Summary Different formulations for the homogenization of elasto-(visco)plastic composites are examined. In the rate-independent case, a robust and generic incremental formulation using algorithmic tangent operators and implicit time-discretization has been developed. For rate-dependent elasto-plasticity, an affine formulation is adopted which transforms the constitutive law into a fictitious thermo-elastic one on which the homogenization is performed.

INTRODUCTION

In this paper, our latest work on the prediction of the macroscopic behavior of two-phase rate-dependent and independent elasto-plastic composites is presented. For this purpose, a two-scale approach is achieved in order to show the influence of the microscopic properties (material characteristics of each phase, shape, volume fraction and orientation of the reinforcements,…). The transition between the microscopic scale, at which we can see the heterogeneity of the material, and the macroscopic one, that of the body, is performed via a mean-field theory, also known as homogenization method. The development of robust algorithms (in term of accuracy, speed and possibility of cyclic and non-proportional loadings) and the issue of the constitutive laws' formulation will be pointed out.

Micro-macro approach and mean-field homogenization models

The main objective is to relate the macro-strain to the macro-stress at a given point $\bar{\mathbf{x}}$. In a micro-macro approach, each macro-point $\bar{\mathbf{x}}$ can be viewed as the center of a representative volume element (RVE). At this scale, the heterogeneity of the material can be observed. Furthermore, it can be shown that if linear boundary conditions are applied to the RVE, then the macroscopic problem is equivalent to that of relating average strains and stresses on the RVE.

Let's consider now a two-phase composite made of inclusions embedded in a matrix. One can define a strain concentration tensor relating the strain averages per phase. Fixing this strain concentration tensor defines a homogenization model. Most of these are based on the fundamental solution of the Eshelby's problem which introduces Eshelby's tensor.

Some well-known homogenization schemes based on Eshelby's result are the Mori-Tanaka scheme, the self-consistent one and interpolative ones. In linear elasticity, once a homogenization scheme is defined with its strain concentration tensor, the macro stiffness can be computed. In the remainder of this paper, we will make use of the Mori-Tanaka model, which is very appropriate for predictions of polymer matrix composites (PMC's) and metal matrix composites (MMC's) with small volume fraction of inclusions.

RATE-INDEPENDENT ELASTO-PLASTIC COMPOSITE MATERIALS

In the case of rate-independent elasto-plastic composites, we can predict the overall behavior either with a *secant operator* ($C^{\sec}$) or a *tangent operator*. The latter can be a *continuum* one (C^{cont}) or an *algorithmic* one ($C^{a\lg}$ - based on a linearization of the return-mapping algorithm):

$$\sigma(x) = C^{\sec}(x) : \varepsilon(x)\ , \quad \dot{\sigma}(x) = C^{cont}(x) : \dot{\varepsilon}(x)\ , \quad \delta\sigma(x) = C^{a\lg}(x) : \delta\varepsilon(x)$$

The Hill-type incremental formulation (which makes use of the tangent operator) has been criticized in the literature for giving too stiff macro responses. However, the secant formulation presents a major disadvantage: no unloading or cyclic loadings are allowed since it assumes a one-to-one correspondence between stress and strain. Keeping this in mind, we tried to improve the predictions of the incremental formulation. Firstly, the use of the continuum operator or the algorithmic one gives significantly different results (the continuum operator gives the stiffest response - proved in [1]). Secondly, numerical experience has shown that good predictions are obtained only when Eshelby's tensor is computed with the isotropic part of the tangent operator. However, in elasto-plasticity, the tangent operator is no longer isotropic and there are several methods for extracting an isotropic part of a fourth-rank tensor. All the other computations are performed with the anisotropic algorithmic modulus. With the incremental method, we have implemented two plasticity models: classical J2 plasticity and Chaboche's model with non-linear kinematic and isotropic hardening. A wide range of experimental confrontations (or with FE if unavailable) can be found in [1] (for spherical reinforcements), and in [2] (for spheroidal inclusions with various but fixed orientations). A very interesting example is the stabilized cyclic tension/compression test illustrated on Figure 1 for aluminum alloy reinforced by SiC short fibers. Recent and yet unpublished results concern distributed-orientation fibers in an elasto-plastic matrix.

RATE-DEPENDENT ELASTO-PLASTIC COMPOSITE MATERIALS

For rate-dependent elasto-plasticity, Hooke's law is still given by: $\dot{\sigma} = E : (\dot{\varepsilon} - \dot{\varepsilon}^{in})$, in which E is Hooke's operator and $\dot{\varepsilon}^{in}$ (inelastic strain rate) is governed by a plastic flow rule. One major difference with rate independent elasto-plasticity is that there is no continuum tangent operator linking stress and strain rates anymore: the homogenization using an *incremental formulation* becomes then impossible, and using it nevertheless with an algorithmic tangent operator leads to predictions which are too stiff. As Masson *et al.* [3] suggested, an alternative is to use a so-called *affine formulation* (basic ideas can be found in [4]). We have investigated the development of such a formulation for elasto-viscoplastic composites and its implementation with a Mori-Tanaka homogenization scheme. Guidelines of this formulation are given hereafter.

For an homogeneous material (e.g.: one of the phases of the composite), constitutive laws can be written such that the inelastic strain rate is governed by a function depending on the stress state and internal parameters, reduced in this case to the accumulated plasticity p, also described by a governing equation:

$$\dot{\varepsilon}^{in}(t) = g(\sigma, p)\,, \quad \dot{p}(t) = h(\sigma, p)$$

Once these equations are defined, we can perform a linearization around time $t = \tau$, at which we are looking for the macroscopic response:

$$\dot{\varepsilon}(t) = \underbrace{S : \dot{\sigma}(t)}_{\dot{\varepsilon}^{el}(t)} + \underbrace{g(\tau) + m(\tau) : [\sigma(t) - \sigma(\tau)] + n(\tau) : [p(t) - p(\tau)]}_{\dot{\varepsilon}^{in}(\tau,t)}$$

$$\dot{p}(t) = h(\tau) + l(\tau) : [\sigma(t) - \sigma(\tau)] + q(\tau)[p(t) - p(\tau)]$$

in which $m(\tau)$, $n(\tau)$ and $l(\tau)$, $q(\tau)$ are the derivatives of g and h w.r.t. σ and p respectively. These derivatives can be evaluated analytically from the governing equations. Solving the equation in p and rearranging terms to introduce the Stieljes-type convolution product (noted $\bullet$), we are left with:

$$\dot{\varepsilon}(t) = [S_\tau \bullet \dot{\sigma}]_{(\tau,t)} + \dot{\varepsilon}^0(\tau,t)$$

in which expressions of $S_\tau(\tau,t)$ and $\dot{\varepsilon}^0(\tau,t)$ can be derived analytically. At this stage, we can note that we are in front of a linear viscoelastic problem with an eigenstrain rate. The macroscopic relation can also be written similarly. Classically, this relation is transformed into the Laplace-Carson domain, in which the Stieljes-type convolution product is reduced to a tensorial contraction:

$$\varepsilon^*(s) = S_\tau^*(\tau,s) : \sigma^*(s) + \varepsilon^{0*}(\tau,s)$$

The relation becomes a fictitious linear thermo-elastic one, valid for each phase, on which the classical homogenization of linear thermo-elastic composites can be applied, as described in [5]. This procedure gives the homogenized (noted with an upper bar) functions $\overline{S}_\tau^*(\tau,s)$ and $\overline{\varepsilon}^{0*}(\tau,s)$, defined in the Laplace domain. A numerical inversion is then achieved to get the equivalent temporal functions. Finally, evaluation of the convolution product appearing in the macroscopic relation of linear viscoelasticity gives the macroscopic solution at time $t = \tau$.

From the algorithmic point of view, an iterative method is used on each time interval $[t_n, \tau]$ because $S_\tau(\tau,t)$ and $\varepsilon^0(\tau,t)$ are functions of the past history of the corresponding phase. Finally, for each time step, an iterative loop is necessary in order to find the right strain increment in each phase, similarly as in the proposed algorithm for elasto-plasticity [2]. Another global iterative loop has to be involved if we are performing a uniaxial tension test given the strain rate in order to get a compatible macro stress tensor.

References

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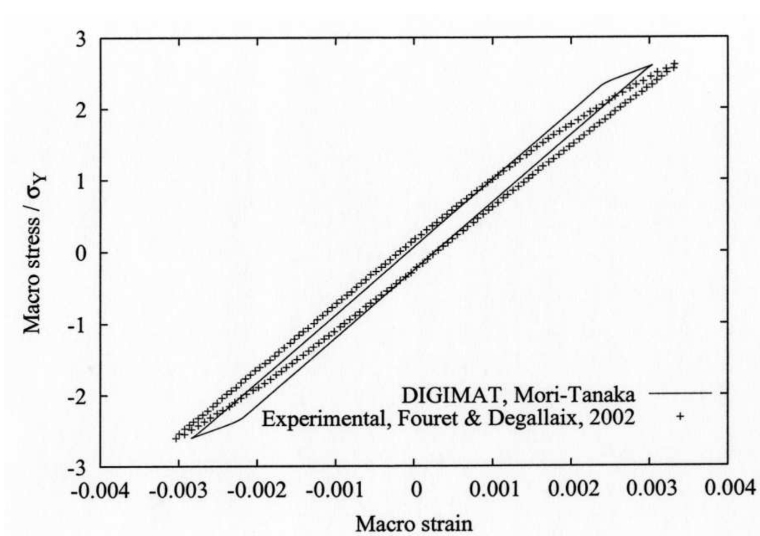

Figure 1