ON APPROXIMATE JACOBIAN MATRICES IN SIMULATION OF MULTIBODY SYSTEMS

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Summary Different forms of approximate Jacobian matrices of equations of motion for stiff multibody systems are under consideration.

INTRODUCTION

It is well known that equations of motion of many multibody models of technical systems are stiff. Solving the corresponding differential or differential-algebraic equations with implicit methods requires evaluations of Jacobian matrix (JM), which is often strong nonlinear in coordinates and velocities. Usually the JM is computed by finite differences, and this process increases several times the CPU expenses for evaluation of the equations [1]. Moreover, use of the JM often increases the profile of the system matrix and makes impossible the application of some effective algorithms such as articulated body method. Evaluation of the JM could be accelerated if analytic expressions for its elements are used [2]. Approximate JM of different types allows further reducing the computational expenses.

EQUATIONS OF MOTION

To simplify the expressions, consider holonomic scleronomic multibody system without closed loops. Let q be coordinates of the system, $q = \left(q_1^T \quad \cdot\cdot \quad q_n^T\right)^T$, where q_j is the $n_j \times 1$ matrix of local coordinates in joint *j*, which define the position of body *j* relative to the previous body i. Consider recursive kinematic relations for 6×1 vectors *V* of linear and angular velocities of adjusted bodies

$$V_j = C_{ij}V_i + S_j\dot{q}_j\,,\; V_j = \left(v_j^T \quad \omega_j^T\right)^T,\; C_{ij} = \begin{pmatrix} I_3 & -\tilde{r}_{ij} \\ 0 & I_3 \end{pmatrix}.$$

Here I_3 is the 3×3 identity matrix, r_{ij} is the vector connecting origins of body-fixed frames *i*, *j*. The recursive relations yield the explicit expressions for velocities

$$V_j = \Phi_j(q)\dot{q}\,,\;\; \Phi_j = C_{0j}\sum_{k\in J(j)}\hat{S}_k^*\,,\;\; \hat{S}_k^* = \left(0 \quad \cdot\cdot \quad 0 \quad C_{0k}^{-1}S_k \quad 0 \quad \cdot\cdot \quad 0\right),$$

where $J(k)$ is the set of indices of joints included in the path from body *k* to body 0. Similar expressions are valid for 6×1 vectors of accelerations $W_j = \Phi_j\ddot{q} + W_j'(q,\dot{q})$.

Equations of motion are derived with the help of the Newton-Euler formalism.

$$f(\ddot{q},\dot{q},q,t) = \sum_{j=1}^{n}\Phi_j^T\left(M_j(q)\ddot{q} + M_j(q)W_j'(q,\dot{q}) + k_j(q,\dot{q}) - f_j(q,\dot{q},t)\right) = 0\,,\qquad M_j = \begin{pmatrix} m_jI_3 & 0 \\ 0 & J_j \end{pmatrix},\; k_j = \begin{pmatrix} 0 \\ \tilde{\omega}_jJ_j\omega_j \end{pmatrix}.$$

Here m_j is the mass and J_j is the inertia tensor of body *j*, the 6×1 matrix f_j contains applied forces and moments reduced to the center of mass. Origins of the body-fixed frames are located in the corresponding centers of mass.

Variation of equations with respect to q and $\dot{q}$ produces the Jacobian matrices

$$\delta f = J^q\delta q + J^v\delta\dot{q}\,.$$

Taking into account the structure of the equations of motion, the JM can be presented by the following parts:

$$J^q = \frac{\partial M\ddot{q}}{\partial q^T} + \frac{\partial k}{\partial q^T} - \frac{\partial Q}{\partial q^T} = J_M^q + J_k^q + J_Q^q\,,\quad J^v = \frac{\partial k}{\partial \dot{q}^T} - \frac{\partial Q}{\partial \dot{q}^T} = J_k^v + J_Q^v\,,$$

where the mass matrix *M* and vectors of the generalized inertia forces *k* and the applied forces *Q* are introduced,

$$M = \sum_{j=1}^{n}\Phi_j^TM_j\Phi_j\,,\quad k = \sum_{j=1}^{n}\Phi_j^T\left(M_jW_j' + k_j\right),\quad Q = \sum_{j=1}^{n}\Phi_j^Tf_j\,.$$

APPROXIMATE JACOBIAN MATICES

The main condition for the approximation of JM consists in assumption that the stiffness of the equations is caused by the applied forces,

$$\left\| J_Q^q \right\| >> \left\| J_M^q \right\|,\ \left\| J_Q^q \right\| >> \left\| J_k^q \right\|,\ \left\| J_Q^v \right\| >> \left\| J_k^v \right\|,$$

or $J^q \approx J_Q^q,\ J^v \approx J_Q^v$.

This assumption is valid if coordinates q change slowly. In particular, partial derivatives of matrices Φ_j are neglected, and variations of velocities and generalized forces are approximately as follows:

$$\delta V_j = \delta\Phi_j \dot{q} + \Phi_j \delta\dot{q} \approx \Phi_j \delta\dot{q}, \qquad (1)$$

$$\delta Q = \sum_{j=1}^{n} \delta\Phi_j^T f_j + \Phi_j^T \delta f_j \approx \sum_{j=1}^{n} \Phi_j^T \delta f_j$$

Let $f_j = \sum f_{jk}$, where the force f_{jk} depends on positions and velocities of interacting bodies j, k,

$$f_{jk} = f_{ij}\left(r_j, r_k, A_{0j}, A_{0k}, v_j, v_k, \omega_j, \omega_k\right).$$

Taking into account assumption (1), the variation δf_{jk} looks like

$$\delta f_{jk} \approx \left(G_{jk1}^r \Phi_j + G_{jk2}^r \Phi_k\right)\delta q + \left(G_{jk1}^v \Phi_j + G_{jk2}^v \Phi_k\right)\delta\dot{q}.$$

The (6×6) *local Jacobian matrices* (LJM) $G_{jk1}^r, G_{jk2}^r, G_{jk1}^v, G_{jk2}^v$ should be computed for every stiff force element. After that the approximate JM of equation of motion can be obtained from the following expressions:

$$J^q \approx -\sum_{j=1}^{n} \Phi_j^T \left(\left(\sum_k G_{jk1}^r \right) \Phi_j + \sum_k G_{jk1}^r \Phi_k \right) = -\sum_{j=1}^{n} \Phi_j^T \left(G_j^r \Phi_j + \sum_k G_{jk1}^r \Phi_k \right),$$

$$J^v \approx -\sum_{j=1}^{n} \Phi_j^T \left(\left(\sum_k G_{jk1}^v \right) \Phi_j + \sum_k G_{jk1}^v \Phi_k \right) = -\sum_{j=1}^{n} \Phi_j^T \left(G_j^v \Phi_j + \sum_k G_{jk1}^v \Phi_k \right), \qquad (2)$$

Evaluation of these matrices is similar to that for the mass matrix.

Some reduction of computational expenses is achieved by simplification of expression for LJM. In fact, the following important relations are often valid for simplified LJM:

$$G_{jk1}^{r*} = G_{jk1}^{v*} = -G_{jk2}^{r*} = -G_{jk2}^{v*} = G_{kj1}^{r*} = G_{kj1}^{v*} = -G_{kj2}^{r*} = -G_{kj2}^{v*} = G_{ji}^{*}, \quad G_{jk1}^{v*} = C_{0j}^T G_{jk1}^v C_{0j} \ldots$$

Moreover, the simplified JM are often symmetric.

BLOCK–DIAGONAL JACOBIAN MATRICES

Expressions for JM obtained in the previous section allow their fast evaluating. Nevertheless, widening of the system matrix profile makes their usage impossible in some cases. For example, consider a system of 1000 bodies interacting due to contact forces. Such system can be used for analyses of a track ballast properties. Mass matrix of the system is block-diagonal whereas the system matrix, which is the sum of the mass matrix and weighted JM, has usually a wide profile so that solving the corresponding equation requires too much CPU time. Numeric experiments show that a very easy and effective solving of this problem consists in the usage of block–diagonal JM, i.e. the non-diagonal blocks G_{jk2}^r, G_{jk2}^v are removed from the expressions (2). The JM assume the form

$$J^q \approx \sum_{j=1}^{n} \Phi_j^T G_j^r \Phi_j, \quad J^v \approx \sum_{j=1}^{n} \Phi_j^T G_j^v \Phi_j.$$

For constraint multibody systems the LJM G_j^r, G_j^v are added to the corresponding local mass matrix M_j. It is not required a separate evaluation both the global mass matrix M and the JM. Consequently, the system matrix can be computed with the help of the composite rigid body algorithm. No widening of the profile is expected because the system matrix obtains the same profile as the mass matrix. Finally, articulated body algorithms can be used as well.

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