GENERALIZED NEWTON – EULER DYNAMIC EQUATIONS

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Summary Novel generalized Newton – Euler dynamic equations are proposed. They are applicable as for rigid so for flexible bodies which kinetic energy is expressed as quadratic form with respect to quasi-velocities. Body mass properties are presented by generalized dense mass-matrices used in finite element discretization and other kinds of mass reduction. Examples of modeling large flexible deflections are solved. The equations solve in general the problem for deriving the dynamic equations of rigid and flexible systems.

INTRODUCTION

Many general-purpose algorithms for computer simulation of multibody systems were created in the recent past [1 – 3]. Modelling of large flexible deflections is a challenging realm of investigations. The main problem is presentation of large node rotations and correct computations of velocity dependent node accelerations, and corresponding inertia forces. Shabana [3] discussed the problem for generalized Newton – Euler equations in dynamics simulation of large flexible deflections. He regarded the velocities of the body fixed coordinate system position and orientation, as well as, of the relative motion of flexible particles. However, the Newton – Euler equations are expressed in terms of the quasi velocities an accelerations and do not concern the kind of parameters at all.

In the paper the inertia forces of a six degree of freedom mass object are defined with respect to quasi-velocities and accelerations. Dense mass-matrices are considered for presentation of the rigid body and flexible element mass properties normally used in finite element theory and different kinds of mass reduction. Generalized Newton – Euler equations are proposed. Numerical results of simulation of large flexible deflections of beam and beam structures verify the precision and effectiveness of the equations proposed.

INERTIA FORCES OF RIGID AND FLEXIBLE MULTIBODY SYSTEMS

The inertia forces in the dynamic equations are derived, according to the Lagrange's equations, differentiating the kinetic energy with respect to the generalized coordinates, velocities and time. Most often the kinetic energy is expressed in terms of the quasi-velocities. For a rigid body *i* they the 6 × 1 matrix-vector $\dot{\boldsymbol{\Delta}}_i = \left[\mathbf{v}_i^{/} \quad \boldsymbol{\omega}_i^{/}\right]^{/}$ compiles the velocity of the body mass centre $\mathbf{v}_i = \left[v_x \quad v_y \quad v_z\right]^{/}$ and the body angular velocity $\boldsymbol{\omega}_i = \left[\omega_x \quad \omega_y \quad \omega_z\right]^{/}$ with respect to inertial frame. Superscript "/" (slash) denotes matrix transpose. For this description of the quasi-velocities the body mass matrix is compiled as follows:

$$\underline{\mathbf{M}}_i = \begin{bmatrix} diag(m_i, m_i, m_i) & \mathbf{0}_3 \\ \mathbf{0}^3 & \boldsymbol{\tau}_i \cdot \mathbf{J}_i \cdot \boldsymbol{\tau}_i^{/} \end{bmatrix}, \qquad (1)$$

where m_i is the mass of the body; $\mathbf{J}_i$ is its inertia tensor; $\mathbf{0}_3$, $\mathbf{0}^3$, $\mathbf{0}_3^3$ are three dimensional matrix row, column and 3×3 square matrix, respectively. Matrix $\boldsymbol{\tau}_i$ is rotation matrix of body *i*. As for a rigid body, similar matrix $\dot{\boldsymbol{\Delta}}_i$ describes the velocities of a node of a flexible element. The translations and rotations of the nodes of flexible elements are dependent, which results in dense (no zero elements) reduced mass matrices. For a 6 × 6 dense mass matrix $\underline{\mathbf{M}}$ (the subscript is missed), selecting of generalized coordinates that define the object position and orientation (for example, coordinates of body mass – centre or a node, and Euler angles) and differentiatiating, according to the Lagrange's equations, of the kinetic energy one obtains the inertia forces **F**. Including the quasi-velocities and accelerations in the expressions so obtained the following generalized Newton – Euler equations are derived:

$$\underline{\mathbf{F}} = \underline{\mathbf{M}} \cdot \begin{bmatrix} \dot{\mathbf{v}} \\ \dot{\boldsymbol{\omega}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\omega}^{\times} & \mathbf{0}_3^3 \\ \mathbf{v}^{\times} & \boldsymbol{\omega}^{\times} \end{bmatrix} \cdot \underline{\mathbf{M}} \cdot \begin{bmatrix} \mathbf{v} \\ \boldsymbol{\omega} \end{bmatrix} - \underline{\mathbf{M}} \cdot \begin{bmatrix} \boldsymbol{\omega}^{\times} \cdot \mathbf{v} \\ \mathbf{0}_3 \end{bmatrix}, \qquad (2)$$

where the superscript "$^{\times}$" denotes skew – symmetric matrix of a three component vector. The reader could verify that replacing matrix $\underline{\mathbf{M}}$ in Eq. 2 by $\underline{\mathbf{M}}_i$ in Eq. 1 one obtains the equations of Newton – Euler for rigid body. The generalized Newton – Euler equations for plane motion are:

$$\underline{\mathbf{F}} = \underline{\mathbf{M}} \cdot \begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{\omega}_z \end{bmatrix} + \begin{bmatrix} 0 & -\omega_z & 0 \\ \omega_z & 0 & 0 \\ -v_y & v_x & 0 \end{bmatrix} \cdot \underline{\mathbf{M}} \cdot \begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix} + \underline{\mathbf{M}} \cdot \begin{bmatrix} \omega_z \cdot v_y \\ -\omega_z \cdot v_x \\ 0 \end{bmatrix}. \tag{3}$$

For a flexible system discretized using finite elements with n nodes the generalized Newton – Euler equations, respectively, the inertia forces in the nodes are

$$\mathbf{F} = \mathbf{M} \cdot \ddot{\mathbf{\Delta}} + \dot{\mathbf{\Delta}}^{\otimes} \cdot \mathbf{M} \cdot \dot{\mathbf{\Delta}} - \mathbf{M} \cdot \dot{\mathbf{\Theta}}, \tag{4}$$

where $\dot{\mathbf{\Delta}} = \left[\dot{\mathbf{\Delta}}_1^{/} \quad \dot{\mathbf{\Delta}}_2^{/} \quad \cdots \quad \dot{\mathbf{\Delta}}_n^{/}\right]^{/}$ and $\dot{\mathbf{\Delta}}_i = \left[\mathbf{v}_i^{/} \quad \boldsymbol{\omega}_i^{/}\right]^{/}$ are the velocities of node i, $\dot{\mathbf{\Delta}}^{\otimes} = diag\left(\dot{\mathbf{\Delta}}_1^{\otimes}, \dot{\mathbf{\Delta}}_2^{\otimes}, \cdots, \dot{\mathbf{\Delta}}_n^{\otimes}\right)$, $\dot{\mathbf{\Delta}}_i^{\otimes} = \begin{bmatrix} \boldsymbol{\omega}_i^{\times} & \mathbf{0}_3^3 \\ \mathbf{v}_i^{\times} & \boldsymbol{\omega}_i^{\times} \end{bmatrix}$, $\dot{\mathbf{\Theta}} = \left[\dot{\mathbf{\Theta}}_1^{/} \quad \dot{\mathbf{\Theta}}_2^{/} \quad \cdots \quad \dot{\mathbf{\Theta}}_n^{/}\right]^{/}$, $\dot{\mathbf{\Theta}}_i = \left[\left(\boldsymbol{\omega}_i^{\times} \cdot \mathbf{v}_i\right)^{/} \quad \mathbf{0}^3\right]^{/}$.

The external and elastic forces are included in the dynamic equations using principle of the virtual work.

EXAMPLE

The generalized Newton – Euler equations are examined solving the example of Kane's rotating slender beam [4]. The results of the numerical experiment show excellent agreement with the basic test.

In Fig. 1 snapshots of motion simulation of falling slender ring over a rigid surface is presented. The segment that breaks the ring line displays the rotation. The dynamic equations are derived using the Lagrange's equations, as well as, the equations proposed in the paper. The numerical results are similar. The data for the size and material of the ring are (all measures are in SI UNITS): radius of the ring – 2.3917; modulus of elasticity $E = 7. \cdot 10^{10}$; cross section area S = 0.0018; moment of inertia of the cross section $I = 1.2151 \cdot 10^{-8}$; mass density ρ = 2690. The initial position of its center at time t = 0 is shown in the figure. The ring rotates with angular velocity $\omega = 1$. Friction is taken into account – $\mu = 0.6$.

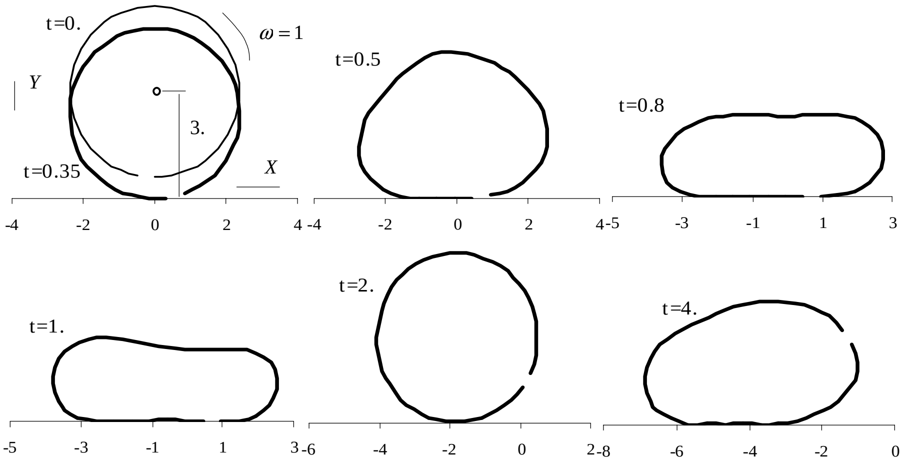

Figure 1. Impact and large deflections of slender ring

CONCLUSIONS

Novel generalized Newton – Euler dynamic equations are derived. Similarly to the rigid body dynamics they provide the basis for development of general recursive algorithms for dynamics modelling of mixed rigid and flexible multibody systems. The equations are consistent with any commercially available computer programs for finite element discretization.

References

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