Hierarchical Tracking Control of Wheeled Mobil Robot

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Summary : Tracking control for autonomous vehicles or mobile robots plays an essential role in the exploration of hazardous areas or planets, such as Mars. If the motion is realized through no-sliding wheels, the problem associated with nonholonomic constraints naturally arise. Based on Jourdain's variational equation, it is possible to formulate the dynamics of such system in terms of the privileged coordinates. Given the desired trajectory, the corresponding reduced Appell's equation can be used to design the control law for the privileged coordinates. On the other hand, to track the non-privileged coordinates, the conditions of constraints are re-structured from which the compensations for the desired values of privileged coordinates are computed. From the simulation results, it is shown that such hierarchical tracking control strategy which simultaneously takes kinematics and dynamics into consideration indeed gives rise to an effective algorithm for tracking problem.

Problem Description

The non-integrability of the nonholonomic constraints makes the problem of tracking control of mobile robot with rolling-without-sliding wheels very difficult to be managed. In the literature, cf. [1], some approaches are based on the kinematic equations and the dynamical equations of the system are not taken into consideration. Such methods ignore the mass and the moment of inertia of the system and thus the designs are deemed impractical. Some of the other methods combine both sets of kinematic and dynamic equations together and consider either enlarged or reduced set of equations to find the control law. However, the former case raises the complexity of the design, while the latter may be subject to under-actuated problem. It is then desired to develop a feasible and practical methodology of controller design which appropriately accommodates the dynamics and the kinematic constraints. The hierarchical tracking control algorithm proposed in this paper suits such need.

The practical problem to be solved in this paper is the tracking of a desired trajectory for a three-wheeled mobile robot, cf. [3], whose configuration is depicted in Figure 1, moving on a horizontal plane. The system may be modeled by a rigid body interconnected with two rolling-without-sliding wheels, cf. Figure 2. The castor wheel is ignored due to its negligible effects on the dynamics of the system. The motion of the wheels may be described by $(x_j, y_j, \psi_j, \theta_j)$, $j = 1, 2$, where (x_j, y_j) denote the coordinates of the center of mass and ψ_j, θ_j are the rotation angle and the orientation angle of the wheels, respectively. The body may be characterized by (x_3, y_3, θ_3), where θ_3 denotes the heading angle of the vehicle. There are total of 11 variables which are subject to five holonomic constraints: (1) $\theta_1 = \theta_2 \equiv \theta$, (2) $\theta_2 = \theta_3 \equiv \theta$, (3) $y_3 = y_1 - b\cos\theta + d\sin\theta$, (4) $x_3 = x_1 + b\sin\theta + d\cos\theta$, (5) $r\dot{\varphi}_2 = r\dot{\varphi}_1 + 2b\dot{\theta}$; and four nonholonomic constraints: (6) $\dot{x}_1 = r\dot{\varphi}_1\cos\theta$, (7) $\dot{y}_1 = r\dot{\varphi}_1\sin\theta$, (8) $\dot{x}_2 = r\dot{\varphi}_2\cos\theta$, (9) $\dot{y}_2 = r\dot{\varphi}_2\sin\theta$. It is assumed further that the two wheels are rotated by motors which generate torques τ_1, τ_2, respectively. The objective of the design is to obtain a controller which enables the system to track a reference trajectory (x_{jr}(t), y_{jr}(t), $\psi_{jr}(t), \theta_{jr}(t), j = 1, 2, x_{3r}(t), y_{3r}(t), \theta_{3r}(t)$) which satisfies the above-mentioned geometric and kinematic constraints.

The Methodology

Based on Jourdain's variational equation and Appell's approach, cf. [2], one may choose ψ_1, θ as privileged coordinates, and derive the reduced Appell's equations of motion as follows:

$$\begin{cases} (m_c + 2m_w + 2m')r^2\ddot{\varphi}_1 + (m_c + 2m_w + 2m')br\ddot{\theta} - m_c dr\dot{\theta}^2 = \tau_1 + \tau_2, \\ (m_c + 2m_w + 2m')br\ddot{\varphi}_1 + [m_c(d^2 + b^2) + \frac{1}{3}m_c'(w^2 + \ell^2) + m'(r^2 + 4b^2) + 4m_w b^2]\ddot{\theta} + m_c dr\dot{\varphi}_1\dot{\theta} = \frac{2b}{r}\tau_2, \end{cases}$$

where m_c, m_w, m' denotes the masses of the components of the system. It is noted that these two equations are decoupled from the non-privileged coordinates, and the controller can be designed independently to track $(\psi_1(t), \theta(t))$. Various methods can be applied to obtain such a controller. Here, the adaptive sliding mode controller is adopted due to its capability of dealing with parameter uncertainties.

While the privileged coordinates can be drived to the desired values without too much effort, the non-privileged coordinates may deviate from the reference values significantly if the initial conditions are not set appropriately or there are some disturbances during the motion. To solve this problem, it is noted from the practical experience of driving that we may change the reference values of the privileged coordinates to steer the non-privileged coordinates. The compensated values for the privileged coordinates may be computed from the kinematic equations: (6), (7), (8') $\dot{x}_2 = r\dot{\varphi}_1 \cos\theta + 2b\dot{\theta}\cos\theta$, (9') $\dot{y}_2 = r\dot{\varphi}_1 \sin\theta + 2b\dot{\theta}\sin\theta$, and those for the reference trajectory. Let $\mathbf{z}_2 = (x_1, y_2)$, $\mathbf{z}_2^e = (x_{1r} - x_1, y_{2r} - y_2)$, $\mathbf{v}_r = (\dot{\varphi}_{1r}, \dot{\theta}_r)$, $\mathbf{y} = (\varphi_1, \theta)$, $\mathbf{y}_r = (\varphi_{1r}, \theta_r)$, and

$$\mathbf{B}_2(\mathbf{y}) = \begin{bmatrix} r\cos\theta & 0 \\ r\sin\theta & 2b\sin\theta \end{bmatrix}, \quad \mathbf{B}_2^r(\mathbf{y}_r) = \begin{bmatrix} r\cos\theta_r & 0 \\ r\sin\theta_r & 2b\sin\theta_r \end{bmatrix}, \quad \boldsymbol{\lambda} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}.$$

It is found that the corresponding compensation is $(\delta\varphi_1, \delta\theta)^T = (\mathbf{B}_2^{-1}(\mathbf{y})(\mathbf{B}_2^r(\mathbf{y}_r)\mathbf{v}_r + \boldsymbol{\lambda}\mathbf{z}_2^e) - \mathbf{v}_r)\Delta t$, where $\boldsymbol{\lambda}$ is used to specify the rate of convergence. From these data, a new set of reference values for the privileged coordinates is computed. The adaptive sliding mode controller mentioned above is then invoked to track the new reference, which in turn drives the non-privileged coordinates to the desired values. It can be shown that with such hierarchical design, all the variables can be steered to the desired value asymptotically.

Simulation Result

Consider the wheeled mobile robot with the following parameters: m_w = 1, m_c = 20, b = 0.5, d = 0.25, r = 0.1. Simulations are conducted to evaluate the performance of the proposed methodology. As shown in Figure 3 and 4, the motion of the wheeled mobile robot can track the desired trajectory (including the heading) successfully, even if the initial configuration is away from the desired configuration. The results demonstrate the effectiveness of the proposed hierarchical control scheme which may be extended to general reducible mechanical systems.

References

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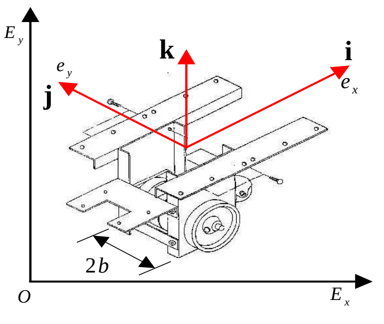

Figure 1

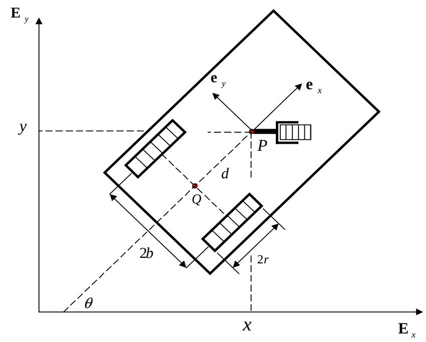

Figure 2

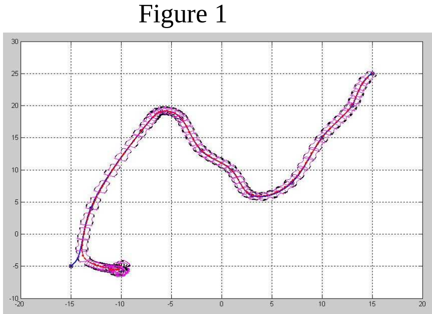

Figure 3

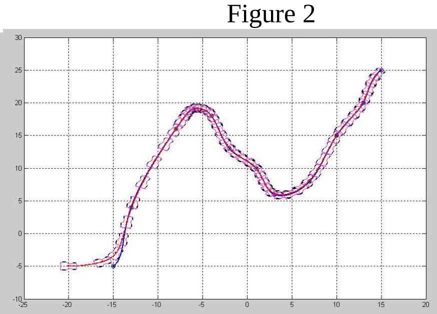

Figure 4