A FLUID INCLUSION IN A POROELASTIC SOLID WITH VOID COMPACTION

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Summary The paper presents a study of the pressure development in a fluid inclusion located within a fluid-saturated poroelastic medium that can exhibit effects of void compaction. The void compaction introduces alterations in the deformability and fluid transport properties of the porous skeleton. A computational approach is used to examine the pressure rise in the fluid inclusion when the poroelastic medium is subjected to a far-field radial compression. The influence of void compaction on the magnitude and decay rate of the fluid pressure is illustrated through the computational simulation.

INTRODUCTION

The classical theory of poroelasticity proposed by Biot [1] has been widely applied for the study of the mechanics of fluid saturated porous media, dealing with soils, rocks, and other multiphase materials. An account of historical and recent developments in poroelasticity are given by de Boer [2] and literature surveys and recent advances in the subject are given by Selvadurai [3], Ehlers and Bluhm [4] and Auriault et al. [5]. The texts by Coussy [6] and Lewis and Schrefler [7] present, respectively, accounts of constitutive modelling and computational advances concerning the subject. The assumptions of the classical theory present limitations to the application of poroelasticity model to situations where the medium can exhibit pore reduction through processes such as void compaction. Void compaction can occur as result of either *reversible* or *irreversible* deformations of the porous skeleton. In the latter instance, the theory of poromechanics must take into consideration both elastic and plastic deformations of the porous skeleton and accompanying change in its fluid transport characteristics. There are, however, examples involving geomaterials such as soft rocks, where the void compaction is predominantly elastic in nature. Results of experiments conducted by Zhu and Wong [8] suggest that void compaction can take place in materials such as sandstone leading to alterations in the elasticity characteristics and the hydraulic conductivity properties. This type of porous skeletal behaviour can be modelled by appeal to the concept of a stress-dependent skeletal response, in terms of the evolution of both the elasticity and fluid transport characteristics. This paper deals with the computational modelling of the problem of a void compaction-susceptible poroelastic geomaterial solid that is bounded internally by a spherical fluid inclusion. The stress dependent alterations of the elasticity and hydraulic conductivity values are determined through an evaluation of the experimental data for *Berea Sandstone* given by Zhu and Wong [8] (Figure 1). The paper investigates the development of the fluid pressure in the spherical fluid inclusion due to the application of a far field uniform radial stress field σ_0. The computational results illustrate the influence of the void compaction process on both the amplification of the pressure in the fluid inclusion, which represents the Mandel-Cryer effect, and on the rate at which dissipation of the fluid pressure takes place.

EQUATIONS OF POROELASTICITY

Combining the governing equations of quasi-static equilibrium for the fluid saturated porous medium, Darcy's law for fluid transport, the constitutive equation for the porous skeleton and mass conservation, we obtain the following field equations for the displacement vector **u** and the scalar pore fluid pressure p:

$$\mu\,\nabla^2\mathbf{u}+\frac{\mu}{(1-2\nu)}\nabla(\nabla.\mathbf{u})+\alpha\,\nabla p=\mathbf{0} \quad ; \quad \frac{k}{\gamma_w}\beta\,\nabla^2 p-\frac{\partial p}{\partial t}+\alpha\beta\frac{\partial}{\partial t}(\nabla.\mathbf{u})=0 \qquad (1)$$

where ∇^2 is Laplace's operator, μ is the linear elastic shear modulus, ν is Poisson's ratio, k is the hydraulic conductivity, γ_w is the unit weight of the fluid, the parameters α and β are given by

$$\alpha=\frac{3(\nu_u-\nu)}{\widetilde{B}(1-2\nu)(1+\nu_u)} \quad ; \quad \beta=\frac{2\mu(1-2\nu)(1+\nu_u)^2}{9(\nu_u-\nu)(1-2\nu_u)} \qquad (2)$$

where $\widetilde{B}$ is Skempton's pore pressure parameter and ν_u is the undrained value of Poisson's ratio. The above equations are quite general, in the sense that appropriate reductions to the behaviour of a poroelastic material saturated with an incompressible pore fluid can be recovered from these expressions. To complete the description of an initial boundary value problem, it is necessary to prescribe boundary conditions and initial conditions for the dependent variables. The boundary conditions can be identified in terms of the Dirichlet, Neumann and Robin-type classical relationships. Also, the uniqueness theorem for the initial boundary value problem in classical poroelasticity is available in the literature.
The influence of void compaction is examined by using stress dependent variations in the elastic modulus and hydraulic conductivity of the poroelastic medium, as deduced from the experimental data given by Zhu and Wong [8]. These variations can be represented in the form

$$E = E_0\left[1 + 2.89\times10^{-3}(\sigma_3/p_a)^2\right] \quad ; \quad k = k_0\left[1 + 4360\xi_d^2\right]\left[1 + 1.6\times10^{-6}(\sigma_3/p_a)^2\right]^{-1} \qquad (3)$$

where E_0 and k_0 are, respectively, the elastic modulus and hydraulic conductivity of the poroelastic material in the uncompacted state, σ_3 is the isotropic stress at which void compaction is induced, ξ_d is the effective shear strain and p_a is the normalizing atmospheric pressure at standard temperature. Poisson's ratio is assumed to be independent of the void compaction process.

NUMERICAL RESULTS

The influence of the void compaction as modelled by the alterations in the elastic modulus and hydraulic conductivity can be incorporated in a conventional Galerkin finite element [6] procedure for the solution of the system of equations (1). The details of the procedure will be omitted, for conciseness. The computational code developed in connection with the research is capable of examining both void compaction and damage initiation in the porous skeleton. The initial parameters for the elastic modulus, Poisson's ratio and hydraulic conductivity of the Berea Sandstone are taken as follows: $E_0 = 5000MPa$; $\nu = 0.20$ and $k_0 = 10^{-13}\ m/\sec$. The fluid inclusion is assumed to be incompressible. The poroelastic solid containing the fluid inclusion is subjected to a far field radial stress σ_0 in the form of a Heaviside step function of time. Figure 2 illustrates the time-dependent variation in the fluid pressure developed in the fluid inclusion for the specific case where $\sigma_0 / E_0 = 0.02$. The computational modelling first deals with the classical poroelasticity result where the elasticity and hydraulic conductivity properties of the poroelastic solid are unaffected by void compaction processes. In the second idealization void compaction induces alterations only in the elasticity properties. The third accounts for an increase in the elasticity modulus and a reduction in the hydraulic conductivity properties.

CONCLUSIONS

The conventional model of poroelasticity can be extended to include effects of void compaction where the compaction process leads to alterations in the elastic deformability and hydraulic conductivity properties. When the alterations are reversible and stress-dependent, the classical approaches to computational modelling of poroelastic media can be conveniently adopted to examine the void compaction effect. The results presented indicate that the void compaction process has only a marginal influence on the amplification of the fluid pressure in the inclusion but has a significant influence on the pore pressure decay processes. The reduction in the hydraulic conductivity can lead to an increase in the time taken for the decay and can lead to the persistence of the generated pressures in the fluid inclusion.

References

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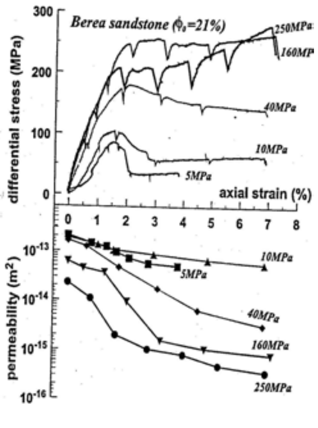

Figure 1. Experimental observations for void collapse in Berea Sandstone (After Zhu and Wong, [8])

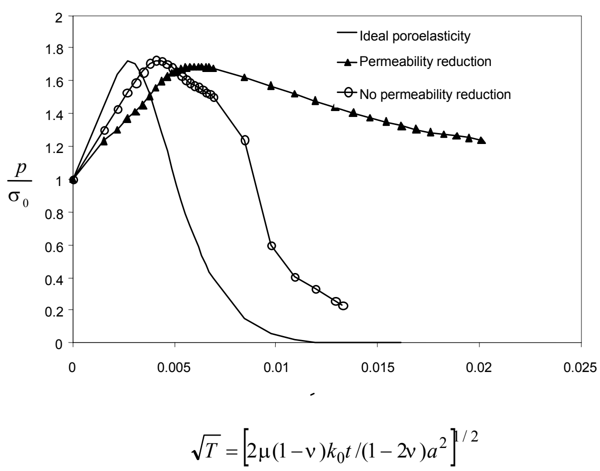

Figure 2. The fluid inclusion located in a proelastic sphere susceptible to the void.