THERMOELASTOPLASTIC BEHAVIOR OF DISCONTINUOUSLY-REINFORCED COMPOSITES CONSIDERING REINFORCEMENT DAMAGE

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Summary In this paper, an incremental constitutive equation of the particle and short-fiber reinforced composites with progressive cracking damage of the reinforcement has been developed considering the temperature change and elastoplasticity. By modifying the load carrying capacity of the damaged reinforcement, the constitutive equation can describe the cracking damage and the debonding damage of the reinforcement as well as the perfect composite.

INTRODUCTION

Analyzing the thermal and/or mechanical behaviors of the composite materials is an important aspect. Eshelby [1] solved the problem of an ellipsoidal inhomogeneity in an infinite body. Mori and Tanaka [2] proposed their concept of the mean stress of the matrix and extended the Eshelby's solution in order to be valid for large number of reinforcements. Tohgo and Chou [3] proposed an incremental theory for a particle-reinforced composite under mechanical load. Asakawa et al. [4] modified the incremental theory [3] and proposed a constitutive equation applicable for both the mechanical and thermal loading. When a reinforcement is cracked, its load carrying capacity is reduced, Cho and Tohgo [5] studied the elastic stress distribution and load carrying capacity of intact and broken ellipsoidal inhomogeneities embedded in an infinite body. Moreover, Tohgo and Cho [6] proposed an incremental theory accounted for the cracking damage.

The present paper presents a thermo-elasto-plastic incremental constitutive equation of short-fiber reinforced composites considering the cracking of the reinforcement and the temperature change as well as the elastoplasticity. Reinforcement damage is incorporated using the Weibull approach to particle fracture. This incremental constitutive equation is a modification of the incremental theory developed by Tohgo and Cho [6]. The difference between the present work and [6] is that the temperature change is considered here in order to use the constitutive equation for the composite structures subjected to thermal and/or mechanical loading conditions. Moreover, for the sake of completeness, the high-order terms of increments, which neglected in [6], are considered during the analysis.

INCREMENTAL CONSTITUTIVE EQUATION

Following Eshelby's principle and Mori-Tanaka's concept, the incremental stress in the particle $d\sigma^p$ is given by:

$$d\sigma^p = d\sigma + d\tilde{\sigma} + d\sigma_1^{pt} = L_1(d\varepsilon_0 - N_1 dT + d\tilde{\varepsilon} + d\varepsilon_1^{pt}) = L_0(d\varepsilon_0 - N_0 dT + d\tilde{\varepsilon} + d\varepsilon_1^{pt} - d\varepsilon_1^*) \quad (1)$$

Moreover, Eshelby's equivalence principle for the cracked reinforcement can be written as:

$$kd\sigma^p = d\sigma + d\tilde{\sigma} + d\sigma_2^{pt} = kL_1(d\varepsilon_0 - N_1 dT + d\tilde{\varepsilon} + d\varepsilon_1^{pt}) = L_0(d\varepsilon_0 - N_0 dT + d\tilde{\varepsilon} + d\varepsilon_2^{pt} - d\varepsilon_2^*) \quad (2)$$

Furthermore, for the reinforcement in the cracking process, the next equation is obtained

$$-(I-k)\sigma^p = d\sigma + d\tilde{\sigma} + \sigma_3^{pt} = L_0(d\varepsilon_0 - N_0 dT + d\tilde{\varepsilon} + \varepsilon_3^{pt} - \varepsilon_3^*) \quad (3)$$

where the definitions of all symbols are omitted here but exist in [6].

The incremental overall strain-stress relation of the composite is obtained by solving Eqs. (1), (2) and (3) as

$$\begin{aligned} d\varepsilon = &\left\{(I+D_1) - [D_1(S-I)-I]H^{-1}dfp[(S-I)D_1 - I]\right\}L_0^{-1}d\sigma + [D_1(S-I)-I]H^{-1}dfpL_0^{-1}(I-k)\sigma^p \\ &-\left\{D_2 + [D_1(S-I)-I]H^{-1}dfp[\alpha_0 I - (S-I)D_2] - \alpha_0\left((I+D_1) - [D_1(S-I)-I]H^{-1}[(S-I)D_1 - I]dfp\right)\right\}dT\delta_{ij} \end{aligned} \quad (4)$$

In Eq. (4), the first, second and third terms represent the strain due to the stress, the cracking damage and the temperature change, respectively. Therefore, the coefficient of thermal expansion can be extracted from the third term.

The incremental average stress of the matrix, $d\sigma^m = d\sigma + d\tilde{\sigma}$, is given by

$$\begin{aligned} d\sigma^m = L_0(I-S)&\left\{\begin{pmatrix}(I+D_1) - [D_1(S-I)-I]H^{-1} \\ \mathbf{x}[(S-I)D_1 - I]df_p + (I-S)^{-1}S\end{pmatrix}L_0^{-1}d\sigma + [D_1(S-I)-I]H^{-1}df_p L_0^{-1}(I-k)\sigma^p\right. \\ &\left.-\left(D_2 + [D_1(S-I)-I]H^{-1}df_p[\alpha_0 I - (S-I)D_2] + \alpha_0\left((I+D_1) - [D_1(S-I)-I]H^{-1}[(S-I)D_1 - I]df_p - I\right)\right)\right\}dT\delta_{ij} \end{aligned} \quad (5)$$

Moreover, the incremental average stresses of the intact and damaged reinforcements are

$$d\sigma^p = d\sigma + d\tilde{\sigma} + d\sigma_1^{pt}, \quad d\sigma^p = d\sigma^m + L_0(S-I)d\varepsilon_1^* \quad \text{and} \quad d\sigma^d = kd\sigma^p \quad (6)$$

The incremental average strains of the matrix and the intact and damaged reinforcements can be evaluated from

$$d\varepsilon^m = L_0^{-1}d\sigma^m, \quad d\varepsilon^p = L_1^{-1}d\sigma^p \quad \text{and} \quad d\varepsilon^d = d\varepsilon_0 + d\tilde{\varepsilon} + d\varepsilon_2^{pt} \quad (7)$$

The following Weibull distribution is adopted for the cumulative probability of the fracture of reinforcement:

$$P_v(\sigma_{max}^p) = 1 - exp\left[-\left(\frac{\sigma_{max}^p}{S_0}\right)^m\right] \quad (8)$$

RESULTS AND DISCUSSION

Firstly, a comparison with other approaches such as the rule of mixture (ROM) and others is made on the effective coefficient of thermal expansion for initial particle volume fraction up to 30% as shown in Fig. 1. It can be seen that the present constitutive equation almost predicts the same values as Wakashima.

The coefficient of thermal expansion normalized by that of the matrix material, α / α_0 , is shown in Fig. 2 as a function of the reinforcement volume fraction for various aspect ratios (λ) of the reinforcement. The coefficient of thermal expansion normalized by that of the matrix (normalized CTE) is higher as the aspect ratio and volume fraction of the reinforcement are lower.

The effect of the temperature on the normalized CTE is shown in Fig. 3. The normalized CTE decreases with the increase of the temperature. For the perfect composite (**k**=**I**), the normalized CTE has constant value at each temperature because all reinforcements are intact. However the results of the cracking damage (**k**=$\mathbf{k}_{cracking}$) are near to those of the debonding damage (**k**=**0**), they lie between those of the perfect composite and debonding damage.

The variation of stresses in the composite, particle and matrix, σ_z, $\sigma^p{}_z$ and $\sigma^m{}_z$, and the damaged reinforcement volume fraction, f_d, are shown in Fig. 4 as function of the composite strain ϵ_z for the three damage modes. It can be seen that the perfect composites exhibit the highest stress-strain relation while the composite with debonding damage exhibits the lowest one. Moreover, the volume fraction of the damaged reinforcement increases with the increase of the strain.

CONCLUSIONS

An incremental constitutive equation is developed for the discontinuously-reinforced composites subjected to mechanical and/or thermal loading conditions and elastic-plastic deformation of the matrix. Also, it can describe the perfect composite and composites with cracking and debonding damage.

The numerical results show that the normalized CTE depends on the volume fraction and aspect ratio of the reinforcement, temperature and the damage mode. For composite material with progressive cracking damage, the perfect composite and composite with debonding damage give the upper and lower bounds of both the CTE and the elastic-plastic behavior.

References

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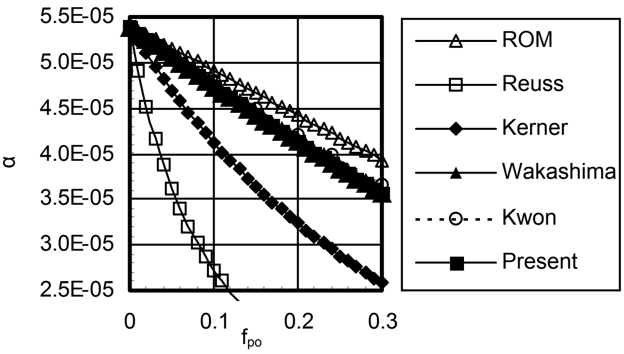

Fig. 1. The effective coefficient of thermal expansion.

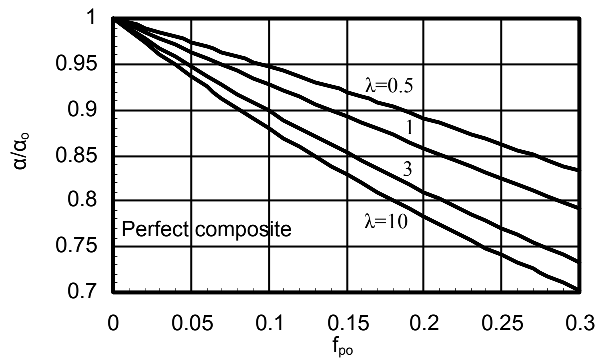

Fig. 2. α/α_o for λ=0.5, 1, 3 and 10.

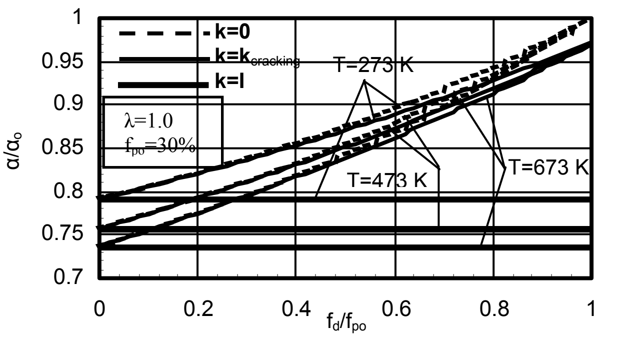

Fig. 3. Effect of **k** and T on α/α_o.

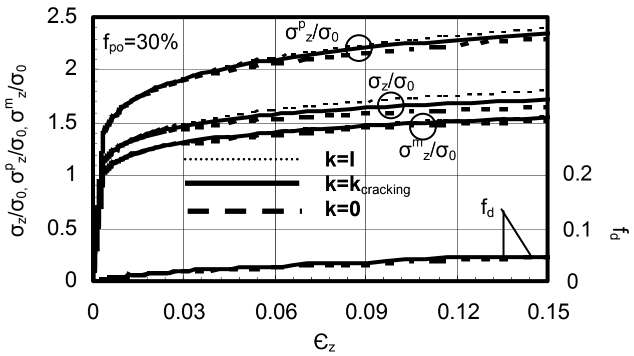

Fig. 4. Elasto-plastic behavior.