

ANALYTICAL MODELS FOR STRESS AND FAILURE ANALYSIS OF NOTCHED HYBRID COMPOSITES

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Summary The design of notched high-performance composites with fibre and textile reinforcement requires a special stress analysis and a proof of their notched strength, which includes the structural parameters of the component combination, material orientation and layer arrangement. For the description of the notch stress behaviour of hybrid constructions, analytical calculation methods are developed at the Institut für Leichtbau und Kunststofftechnik (ILK). The model presented here is based on the enhanced laminate theory combined with the method of complex-valued displacement functions and the method of conformal mappings, from which adapted approaches for the stresses and displacements can be obtained. Extensive experimental research is also conducted. For developing of more accurate failure models an improved stress analysis of notched anisotropic plates combined with a physically based strength analysis is used. The discussed approach is demonstrated here for the example of textile-reinforced timber constructions.

STRESS ANALYSIS

The analytical calculation of notched stresses of anisotropic reinforced composites under mechanical loads has already been conducted by many authors, primarily by means of the method of complex-valued stress functions and the method of conformal mappings. For instance, the notched stress behaviour with tensile/compressive or shear load as well as pure bending load has been treated in [1] dependent on material-specific influence parameters and on the notch contour. Hygrothermal stress concentration effects have been described in [2] and [3], where additional references concerning the general issue of notched fibre composite plates are also provided. On the other hand, only occasionally authors have studied the analytical notch calculation of multi-layered composites because of the coupling of membrane problems and plate bending problems arising in the event of unsymmetrical stacking sequences [4]. Among others, Whitney [5] has demonstrated that in this case ignoring the membrane-bending coupling can lead to appreciable errors of up to 300 %. Therefore, a reliable analysis of stress concentrations in textile-reinforced hybrid composites presupposes the consideration of the complete laminate stiffness matrix.

For a realistic stress concentration analysis, the hybrid composite is modelled as a multilayered plate with a centric cut-out. Since such structures are thin-walled, they can be globally described by means of the classical laminate theory. Furthermore the method of complex-valued displacement functions of anisotropic plates is used. Expanding this method, the solution of the generalised plate equation for multi-layered composites is based on the complex-valued displacement approach for the homogeneous solution and purely mechanical load

$$w_0 = 2 \operatorname{Re} \left(\sum_{k=1}^4 \Psi_k (Z_k) \right),$$

with four analytical functions $\Psi_k(Z_k)$, which are related to four different complex planes $Z_k = x + \mathbf{m}_k y$ ($k = 1 \dots 4$). The complex parameters \mathbf{m}_k are obtained from the characteristic equation of the generalised plate equation. Then the eight roots are always pairwise complex conjugates for actual materials, i. e. $\overline{\mathbf{m}_5} = \mathbf{m}_1$, $\overline{\mathbf{m}_6} = \mathbf{m}_2$, $\overline{\mathbf{m}_7} = \mathbf{m}_3$, $\overline{\mathbf{m}_8} = \mathbf{m}_4$. To set up the boundary conditions for the stress concentration problem, the notch area is expediently reduced to the exterior of the unit circle through a conformal mapping w

$$Z = w(z) = R \left(z + \sum_{k=1}^{\infty} r_k z^{-k} \right) + C,$$

whereby the complex figures R and C result in a rotation-stretching or a translation, respectively. The coefficients r_k of this series have to be defined suitably. For a description of elementary elliptical notches with the semi-axes a and b , e.g., the following equation is sufficient as a conformal mapping:

$$Z = w(z) = \frac{a+b}{2} z + \frac{a-b}{2} \frac{1}{z}.$$

FAILURE CRITERION

Semi-empirical criteria developed in recent years for determination of strength of anisotropic plates with notches are based mainly on rough approximations of the stress concentration decay. Main assumptions are here e.g. polynomial shape of the decay function and transferability of the stress concentrations from an infinite to a finite plate. Correction factors are usually derived from a stress analysis for isotropic materials. Semi-empirical failure models assume that a fibre/matrix composite plate with a notched region fails exactly when the stress concentration at a characteristic distance from the edge of the notch reaches the strength of a notch-free composite. Very often, the experimentally measured characteristic distances are taken to be independent of laminate design, fibre orientation and dimensions of the notches. They are then transferred to a great variety of laminates made of the same type of fibre. The disadvantage of such analysis strategy is that physically inconsistent results are produced for some fibre orientations in highly anisotropic laminates. An improved stress analysis of notched anisotropic plates combined with a physically based strength analysis [2,6] gives the possibility of developing more accurate failure models, which allow a detailed and realistic prediction of the reliability of notched hybrid composite structures.

EXPERIMENTAL VERIFICATION

Extensive load tests are conducted for experimental verification of the developed calculation methods. In particular, the decaying behaviour of the peak stress concentrations is measured and the notched failure is observed with the help of modern 3D field measurement methods such as ESPI (Electronic Speckle Pattern Interferometry) and the grey-value correlation method. The tried and tested strain-gauge technique is applied for reference measurements. Certainly the field measurement methods necessitate a relatively high effort during the implementation and evaluation of the experiment; however, in comparison to the strain-gauge technique they provide not only local values, but details concerning the displacement or strain distribution in the entire field of measurement.

In order to determine direction-dependent characteristic material values, the tension/compression-torsion tests (T/C-T test) were conducted. In the T/C-T tests, the failure-critical stress combinations along pre-stated load paths were introduced (Fig. 1) with the help of a specially enhanced load-controlled multi-axial testing machine with adapted strain-twist extensometers.

It was proved, that the notched strength of composite materials depends additionally on the notch size, e.g. [7]. The notched strength of the selected CFRP composite depending on the radius of the circular notch is presented in Fig. 2. It contains the results obtained using two variants of the point stress criterion (PSC), i.e. according to Whitney-Nuismer and Karlak.

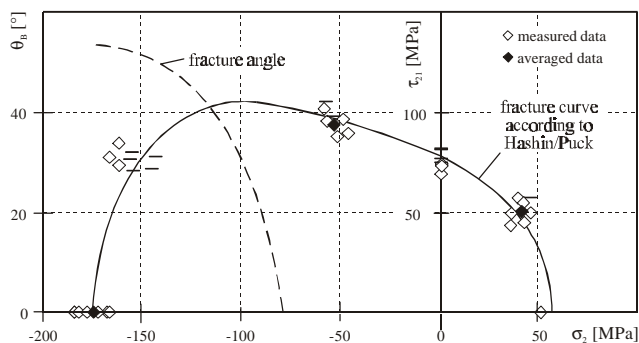


Fig. 1: Strengths, failure curve and fracture angle for tested tube specimens

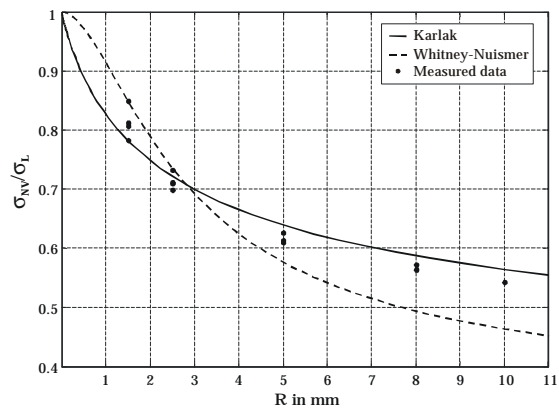


Fig. 2: CFRP notched stress depending on the notch radius

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