CUBICALLY NONLINEAR WAVES IN STRUCTURED MATERIALS OF MACRO-, MICRO- AND NANOSCALE

Carlo Cattani* and Jeremiah Rushchitsky**

** DiFARMA, University of Salerno, Via Ponte Don Melillo, I-84084 Fisciano (SA), Italy*
*** S.P.Timoshenko Institute of Mechanics, Nesterov str. 3, 03680 Kyiv, Ukraine*

Summary The cubically nonlinear waves of displacement are discussed. A medium of propagation is a composite material with fillers of macro-, micro- or nanoscale. Two structural models of elastic deforming process are used – macro-model (classical model with effective physical constants) and micro-nanomodel (nonclassical model of two-phase mixture with absolutely different set of elastic constants). Many new wave effects are theoretically and numerically described and commented.

INTRODUCTION

Last two years authors are actively analyzing the cubically nonlinear waves in elastic materials with internal structure. This analysis can be divided into a few steps. 1. We started with the statement of the problem, for this type of waves was ignored up to now by mechanicians despite of well study of the cubically nonlinear waves in physics. 2. Then some reasons for these waves studying were produced, including simplicity of transition from conventional in mechanics of materials quadratically nonlinear waves to cubically ones and the new possibilities of wave analysis – quadruples (four waves interaction), for instance. 3. Within the framework of cubic nonlinearity, many new wave effects can be predictted and described. These predictions were formulated and commented. 4. Some of new effects were theoretically and numerically analyzed for plane waves. For example, new schemes of wave profile evolution were detected. These steps will be explained in the paper.

STATEMENT OF THE PROBLEM

From pioneer works of 60-ies up to this time, the theory of nonlinear waves in elastic materials is based on Murnaghan quadratic potential and subsequent simplification in result of which the potential is transformed from quadratically nonlinear relative to strain tensor into quadratically nonlinear relative to displacement vector [1,4,5]. This simplification resulted restrictions to the analysis of quadratically nonlinear waves only. The situation was looked as if mechanics of materials forbids the other type of nonlinear waves. But nonlinear physics knows many examples of cubically nonlinear waves, analysis of which is high advanced [1]. Authors paid the attention on the fact that the third harmonics (as a conventional consequence of cubic nonlinearity) appears in the general wave picture in materials and the rejection of mentioned simplification leads to the theory with cubic and higher orders of nonlinearity. Therefore the statement of the problem on cubic nonlinearity and cubically nonlinear waves seemed authors to be rational.

SOME REASONS FOR THESE WAVES STUDYING

The main reason is a pragmatic intention to study the quadruples and different partial wave problems associated with quadruples. But quadruples can be considered within the framework of cubic nonlinearity presence only. This is well explained in nonlinear optics, for example, where is shown in particular that some materials admit cubic nonlinearity and forbid the quadratic one. We shown that elastic materials include both nonlinearities together and the cubic nonlinearity can be displayed separately in some special cases (transverse waves). The next important reason consists in that including the cubically nonlinear waves into the general theory of waves in materials is the fact of a self-development of this theory. This transition from conventional in mechanics of materials quadratically nonlinear waves to cubically ones seems to be simple now. One of more particular reasons is the opening possibility to analyze the non-classical self-switching problem (the classical one is switching from fixed frequency to doubled frequency, the non-classical – to triple frequency)[3].

PREDICTION OF NEW WAVE EFFECTS

Here, the main possibilities are listed, which show in which a way the new wave effects can be predicted and described. The basic nonlinear (quadratically and cubically) system for the model 1 is [1]

$$\rho\, u_{1,tt} - (\lambda + 2\mu)\, u_{1,11} = N_1\, u_{1,11} u_{1,1} + N_2 \left(u_{2,11} u_{2,1} + u_{3,11} u_{3,1} \right) + N_3\, u_{1,11} \left(u_{1,1} \right)^2 + N_4 \left(u_{2,11} u_{2,1} u_{1,1} + u_{3,11} u_{3,1} u_{1,1} \right), \quad (1)$$

$$\rho u_{2,tt} - \mu\, u_{2,11} = N_2 \left(u_{2,11} u_{1,1} + u_{1,11} u_{2,1} \right) + N_4\, u_{2,11} \left(u_{2,1} \right)^2 + N_5\, u_{2,11} \left(u_{1,1} \right)^2 + N_6\, u_{2,11} \left(u_{3,1} \right)^2, \quad (2)$$

$$\rho\, u_{3,tt} - \mu\, u_{3,11} = N_2 \left(u_{3,11} u_{1,1} + u_{1,11} u_{3,1} \right) + N_4\, u_{3,11} \left(u_{3,1} \right)^2 + N_5\, u_{3,11} \left(u_{1,1} \right)^2 + N_6\, u_{3,11} \left(u_{2,1} \right)^2. \quad (3)$$

Possibility 1. The first standard problem of the wave interaction analysis with only quadratic nonlinearity consists in the generation of second harmonics of the plane longitudinal wave when initially the last one is only excited. This problem is described by equation $\rho\, u_{1,tt} - (\lambda + 2\mu)\, u_{1,11} = N_1\, u_{1,11} u_{1,1}$. In this case, quadratically nonlinear term $\left(N_1\, u_{1,11}\, u_{1,1}\right)$ is responsible for the mentioned above effect in the nonlinear wave theory. The cubically nonlinear term $\left(N_3\, u_{1,11}\, (u_{1,1})^2\right)$ in equation (1) will be responsible for the presence of third harmonics generation. Thus the new possibility of analysis of the third harmonics generation not known before is revealed.

Possibility 2. This possibility emerges as the most natural special case of possibility 1. We can analyze separately the influence of the progress of second and third harmonics (in time or in space) on the evolution of the initial profile of the longitudinal wave. Results of this virtual separate analysis can be compared at this stage but later they should be considered together.

Possibility 3. The second standard problem with only quadratic nonlinearity consists in the initial excitation of only transverse wave and further analysis of new longitudinal waves, since new transverse waves cannot be generated. With introduction of cubic nonlinearity this problem obtains new features. The presence of the term $\left(N_4\, u^*_{2,11}\, (u^*_{2,1})^2\right)$ in equation (2) describing the propagation of a transverse wave guarantees that in proposed approach of elastic potential transverse horizontal wave will generate its own third harmonics. It is also important to note that in the classical quadratic approach where the term $\left(N_4\, u^*_{2,11}\, (u^*_{2,1})^2\right)$ is absent, the initially excited transverse wave in the form of the first harmonics doesn't generate other harmonics besides itself, and therefore the second standard problem is not interesting in application to the transverse wave.

Possibility 4. Within the framework of used approach the new problem (let us call it the fourth standard problem) can be formulated. It will consist in the initial excitation of a certain transverse wave (for example, the vertical one) and following analysis of the generated another transverse wave (in this case, the horizontal one). Thus it becomes possible to describe the new energy pumping effect - from one transverse wave to another.

Next two possibilities are associated with the method of slowly varying amplitudes. In this case, we need to return to equations (1)–(3) and apply mentioned above method of slowly varying amplitudes. Here we face many possibilities, two of which we will sketch below. It must be noted here that starting from equations (1)–(3) one can obtain and analyze the shorten and evolution equations for the specific case of cubic nonlinearity only.

Possibility 5. Using the evolution equations it's possible to study the interaction of four waves or the so-called wave quadruple problem. Different possibilities for the choice of four elastic waves can be considered, including the case of "two pumping waves + one signal wave + one idle wave" which is similar to the problem of parametric amplification.

Possibility 6. Starting with the evolution equations that incorporate cubic nonlinearity, we can analyze the new self-switching problem (it is considered above for quadratically nonlinear elastic waves). Within this framework it's possible to describe the mechanism of frequency switching for elastic waves where the frequency switches to three times of its original size and vice versa.

Similar predictions will be formulated for model 2 also (they will be written in more complicated form according to the model complexity). These predictions will be commented.

THEORETICAL AND COMPUTER ANALYSIS OF THE WAVE PICTURE

Some of new effects were theoretically and numerically analyzed for plane waves. Three different methods were used. The method of successive approximations was the basic one for the analysis of problems fixed in Possibilities 1-4. A based on the second approximation classical approach was realized. As an example, the new schemes of wave profile evolution will be shown (the scheme in four steps for quadratically nonlinear waves and in three steps for cubically nonlinear waves). Different composite materials with micro- and nano-fillers will be used in modelling. Using the method of slowly varying amplitudes we obtain additional instrument for analysis of wave interaction – shorten and evolution equations. The main practical problem here is the quadruples problem. For the instance, the problem of two power, one idle and one signal waves will be explained. At least the wavelet based method was applied. It used the introduced by authors elastic wavelets family [2]. An example of theoretical and computer analysis will be concerned solitary plane longitudinal waves. Here the "Mexican hat" wavelet family was successfully used. The phenomenon of decomposition of the initial pulse into two different pulses will be commented.

References

[1] Rushchitsky J.J., Cattani C.: Cubically nonlinear elastic waves: wave equations and methods of analysis. *Intern. Appl.Mech* **39**:1124-1156, 2003.

[2] Rushchitsky J.J., Cattani C.: Solitary elastic waves and elastic wavelets. *Intern.Appl.Mech* **39**:724-736, 2003.

[3] Rushchitsky J.J.: Self-switching of displacement waves in elastic nonlinearly deformed materials. *Comptes Rendus de l'Academie des Sciences, Serie IIb Mecanique.* **330**:175-180, 2002.

[4] Rushchitsky J.J.: Novel solitary waves in elastic materials: existence and nonlinear interaction. *Abstract Book of the ICTAM 2000*, Chicago, USA. - P. 165.

[5] Rushchitsky J.J.: Interaction of waves in solid mixtures. *Applied Mechanics Reviews* **52**: **2,** 35-74, 1999.