

# TRAPPING OF PLASTIC WAVES BY ADIABATIC DEFORMATION

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*Summary* The adiabatic heating due to conversion of plastic work into thermal energy substantially changes the boundary value problem of plastic wave propagation. The thermal coupling in plastic wave propagation leading to adiabatic wave trapping is the main subject of this study. Two cases are analysed, the adiabatic wave trapping in tension and also in shear. The case of shear is relatively new. Theory, experiments and numerical analyses of the Critical Impact Velocity (CIV) in shear is the main part of this contribution. A review of author's recent publications on CIV in tension and shear is presented.

## INTRODUCTION

In the late forties of the last century von Kàrman [1-3] and others developed a theory for propagation of one-dimensional plastic waves in a long bar. It was demonstrated that if a long bar is loaded in tension by a sufficiently high impact velocity, plastic deformation is concentrated near the impact end of the bar. The theory was limited to rate-independent and isothermal case. However, plastic deformation of materials is rate and temperature dependent. The adiabatic heating causes usually a material softening leading to adiabatic wave trapping. Localization of plastic deformation in adiabatic conditions superimposed on inertia effects (waves) causes that the plastic wave speed reaches zero and the Critical Impact Velocity (CIV) is occurring. It is shown that the CIV occurs in both tension and shear. The case of shear has been found and analysed more recently [5-10].

## CRITICAL IMPACT VELOCITIES IN TENSION AND SHEAR

### The Critical Impact Velocity in tension

In the previous analysis of the CIV in tension the adiabatic heating was neglected. A more complete and closer to the reality of fast plastic deformation is assumption of the adiabatic deformation. The equation of heat conduction with internal sources applicable to dynamic plasticity is given by

$$\rho C_p \frac{\partial T}{\partial t} = \beta \sigma(\varepsilon_p, \dot{\varepsilon}, T) \frac{\partial \varepsilon_p}{\partial t} - \lambda \frac{\partial^2 T}{\partial x_1^2} \quad (1)$$

where  $\rho$ ,  $C_p$ ,  $\beta$  and  $\lambda$  are respectively the density, specific heat, Taylor-Quinney coefficient and thermal conductivity,  $\varepsilon_p$  and  $\sigma$  are respectively the plastic strain and true stress, the direction of the heat conduction is  $x_1$ . In fast processes of plastic deformation it can be assumed that the conductivity is zero, then one yields

$$\frac{dT}{d\varepsilon_p} = \frac{\beta}{\rho C_p} \sigma(\varepsilon_p, \dot{\varepsilon}, T) \quad \text{and after integration} \quad \Delta T_A \approx \frac{\beta}{\rho C_p} \int_0^{\varepsilon_{pm}} \sigma[\varepsilon_p, T(\varepsilon_p), \dot{\varepsilon}] d\varepsilon_p \quad (2)$$

where  $\Delta T_A = T - T_0$  is the initial temperature. Here is assumed that the strain rate is a parameter. The standard wave equation is given by

$$\frac{\partial^2 U_1}{\partial t^2} = C^2(\varepsilon) \frac{\partial^2 U_1}{\partial x_1^2} \quad \text{in the elastic range } C_0 = \left( \frac{E}{\rho} \right)^{1/2} \quad \text{and in the plastic range } C_p(\varepsilon_p) = \left( \frac{1}{\rho} \frac{d\sigma}{d\varepsilon_p} \right)^{1/2} \quad (3)$$

where  $C_0$  is the long-bar elastic wave speed in the  $x_1$  direction,  $E$  is the Young's modulus. In the adiabatic conditions the tangent modulus  $(d\sigma/d\varepsilon_p)_A$  versus plastic strain for most of metals and alloys is lower than the isothermal. Therefore, the speed of plastic waves in the adiabatic conditions is lower than in the isothermal case. A simple evaluation of the adiabatic tangent modulus was given in [4]. If constitutive relation  $\sigma(\varepsilon_p, T)_{\dot{\varepsilon}}$  is explicitly known the adiabatic tangent modulus is given by

$$\left( \frac{d\sigma}{d\varepsilon_p} \right)_A = \frac{d}{d\varepsilon_p} [\sigma(\varepsilon_p, T_0)_{\dot{\varepsilon}} - \sigma(\varepsilon_p, T_0 + \Delta T_A(\varepsilon_p), \dot{\varepsilon})] \quad \text{and} \quad C_p(\varepsilon_p)_A = \left[ \frac{1}{\rho} \left( \frac{d\sigma}{d\varepsilon_p} \right)_A \right]^{1/2} \quad (4)$$

The critical impact velocity in tension can be found by applying the method of characteristics [3,5]. The CIV in adiabatic conditions is therefore obtained in the following form

$$V_{CR} = \int_0^{\varepsilon_m} \left[ \frac{1}{\rho} \left( \frac{d\sigma}{d\varepsilon} \right)_A \right] d\varepsilon \quad \text{if } \varepsilon = \varepsilon_m \text{ then } \left( \frac{d\sigma}{d\varepsilon_p} \right)_A = 0 \quad \text{and} \quad C(\varepsilon_m)_A = 0 \quad (5)$$

It is clear that  $\varepsilon_m$  is the instability point of the adiabatic stress vs. total strain curve at predefined value of strain rate  $\dot{\varepsilon}$ . Theoretical values of CIV in tension are close to that found from experiments, for example [3,5]. Values of this material

constant (CIV) vary in the following limits  $60 \text{ m/s} < V_{CR} < 200 \text{ m/s}$ . In single crystals of Al :  $30 \text{ m/s} < V_{CR} < 90 \text{ m/s}$  depending on orientation, [5].

### The Critical Impact Velocity in shear

Localization of plastic deformation during fast shearing by Adiabatic Shear Bands (ASB) is a common failure mode in many materials. If the rate of shearing is sufficiently high the adiabatic process of deformation is superimposed on propagation of plastic waves in shear and it leads to failure near the area of impact. In that case the plastic waves propagate in  $x_1$  direction and displacement is in  $x_2$  direction. The wave equation is given by

$$\frac{\partial U_2}{\partial t^2} = C_2^2(\Gamma) \frac{\partial U_2}{\partial x_2} \quad \text{in the elastic range} \quad C_2 = \left( \frac{\mu}{\rho} \right)^{1/2} \quad \text{and in the plastic range} \quad C_{2p}(\Gamma_p) = \left[ \frac{1}{\rho} \left( \frac{d\tau}{d\Gamma_p} \right) \right]^{1/2} \quad (6)$$

where  $C_2$  is the elastic wave speed in shear,  $C_{2p}$  is the plastic wave speed, and  $\tau$  and  $\Gamma_p$  are respectively the shear stress and plastic shear strain. It should be noted that in this case there is no change of the current surface where the shear stress is imposed and the instability and localization are triggered exclusively by adiabatic softening. The CIV in shear is relatively a new subject of research and was introduced and discussed in [6] and [7]. An analytical model has been developed in [6] and [7], according to that analysis the CIV in shear is given by

$$V_{CR} = C_2 \Gamma_e + \int_{\Gamma_e}^{\Gamma_m} C_{2p}(\Gamma_p) d\Gamma_p \quad (7)$$

where  $\Gamma_e$  is the elastic shear strain at the yield limit and  $\Gamma_m$  is the localisation strain in the *adiabatic* conditions of deformation. It must be remembered that all derivations are for a constant strain rate, characteristic for the process range, that is  $10^3 \text{ 1/s} < \dot{\Gamma} < 10^4 \text{ 1/s}$ . Experiments performed at LPMM with the direct impact Modified Double Shear (MDS) technique, [8], confirmed existence of the CIV in shear for many metals and alloys. Numerical analyses by FE with complete wave propagation scheme and thermal coupling also confirmed existence of CIV in shear for VAR 4340 steel and Ti-6Al-4V alloy, [9,10]. Values of CIV for those alloys determined by FE analyses are respectively 103 m/s and 121 m/s. Details are the subject of presentation.

## CONCLUSIONS

Existence of the Critical Impact Velocities in tension and shear has been confirmed by theoretical, experimental and numerical means. The analysis of CIV in tension has been improved by introduction of thermal coupling in the form of adiabatic heating. The phenomenon of CIV in shear is relatively new. A complete analysis of the CIV in shear with thermal coupling (adiabatic process of deformation) has been performed. It was found that a unique superposition of plastic shear waves and adiabatic softening triggers this phenomenon. Both, CIV in tension and shear can be assumed as specific material constants.

## References

- [1] Kàrman T.: On the propagation of plastic strains in solids, Reports NDRC A-29, OSRD 365, Feb. 1942.
- [2] Duwez P.E., Clark D.S.: An experimental study of the propagation of plastic deformation under conditions of longitudinal impact, Proc. ASTM **47**: 502-532, 1947.
- [3] Kàrman T., Duwez P.E.: The propagation of plastic deformation in solids, J. Appl. Phys., **21**: 987-994, 1950.
- [4] Klepaczko J.R.: Strain rate history effects in Body-Center-Cubic metals, ASTM-STP 765, 251-267, 1982.
- [5] Klepaczko J.R.: Generalized conditions for stability in tension test, Int. J. Mech. Sci., **10**: 297-313, 1968.
- [6] Klepaczko J.R.: Plastic shearing at high and very high strain rates, Proc. Conf. Eurodymat, J. Phys. **IV-4**: C8-C35, 1994.
- [7] Klepaczko J.R.: On the critical impact velocity in plastic shearing, EXPLOMET'95, Proc. Int. Conf. on Metallurgical and Materials Application of Shock-Wave and High-Strain-Rate Phenomena, Elsevier Science, Amsterdam, 413-419, 1995.
- [8] Klepaczko J.R.: An experimental technique for shear testing at high and very high strain rates, the case of mild steel, Int. J. Impact Engng, **15**: 29-39, 1994.
- [9] Klepaczko J.R., Klosak M.: Numerical study of the critical impact velocity in shear, Eur. J. Mech. A/Solids, **18**: 93-113, 1999.
- [10] Bonnet-Lebouvier A.S., Klepaczko J.R.: Numerical study of shear deformation in Ti-6Al-4V at medium and high strain rates, Int. J. Impact Engng, **27**: (2002), 755-769.