

# STRENGTH EVALUATION OF FUNCTIONALLY GRADED MATERIALS UNDER SEVERE THERMAL ENVIRONMENTS

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*Summary* Thermal shock strengths of a functionally graded material plate are analyzed when the plate is suddenly exposed to an environmental medium of different temperature. The critical parameters governing the level of the transient thermal stress in the medium are identified. Thermal shock resistances of the functionally graded materials are analyzed using both the maximum local tensile stress criterion and the maximum stress intensity factor criterion.

## INTRODUCTION

Thermal shock conditions which arise during the sudden heating or cooling of functionally graded materials (FGMs) can result in very high stresses. The degree of damage and strength degradation of the materials subjected to severe fluctuating thermal environments is a major limiting factor in relation to service requirements and lifetime performance. Therefore, it is important to analyze the internal thermal stresses in FGMs and to evaluate their resistance to thermal loading such as thermal shock. The purpose is to control the thermal stress level for a given material system such that it does not damage under a given thermal environment.

This paper presents an approach to the calculation of the thermal stresses and the extent of crack propagation in a functionally graded material plate shown in Fig. 1. Solutions are obtained for the maximum thermal shock that the material can sustain without failure according to the two conditions that: (i) maximum local tensile stress equals the local tensile strength of the material, and (ii) maximum stress intensity factor for representative pre-existing cracks equals the local fracture toughness of the material.

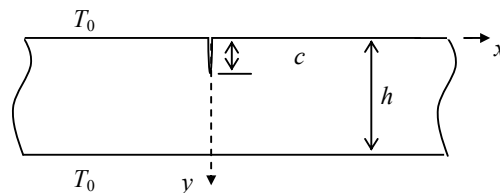


Figure 1. A surface crack in a FGM plate, which is subjected to a sudden thermal load  $T_0$  on its surfaces

## THERMAL SHOCK RESISTANCE OF FGMs

Thermal shock resistance is a major issue in the design of engineering materials for high-temperature applications. A central problem in designing against thermal shock is the identification of an appropriate material selection criterion in order to select the most shock-resistant material for a given application. Generally, there are two distinct criteria for the determination of thermal shock resistance (the admissible value of  $T_0$ , which is denoted as  $T_c$ ):

- (i) A stress-based criterion: It is assumed that the plate has a deterministic strength  $\sigma_{bc}$  on its top surface, where thermal stress has a maximum value. Failure occurs when the maximum thermal stress that appears on the plate surface  $\sigma_{\max}(t)$  attains the value  $\sigma_{bc}$ .
- (ii) A fracture mechanics-based criterion: This criterion assumes that failure occurs when the maximum thermal stress intensity factor  $K_{\max}(c, t)$  attains the fracture toughness of the material  $K_{IC}(c)$ , where  $c$  is the depth of a pre-existing crack and  $t$  is time.

Quantitative analysis has been made for a two-phase TiC-Ni FGM. The volume fraction of ceramic in the FGM is

expressed as a power function of  $y$ :  $V_c=1-(y/h)^g$ , in which  $V_c$  represents the volume fraction of the ceramic phase in the FGM,  $g$  is known as the gradient index. Thus the surfaces at  $y=0$  corresponds to pure ceramic and at  $y=h$  to pure metal. Typical values of the sustainable temperature  $T_c$  as a function of crack length are plotted in Fig. 2, which is calculated for a gradient index  $g=2$ . It is clear that thermal shock resistance predicted by stress-based criterion does not depend on the pre-existing crack. Since the thermal stresses are in self-equilibrium, the stress intensity factors are zero for both an un-cracked plate ( $c=0$ ) and a fully cracked plate ( $c=h$ ). Accordingly, if the fracture mechanics-based criterion is used, the admissible temperature  $T_c$  will be infinity both at  $c=0$  and  $c=h$ .  $T_c$  decreases with crack depth from infinity to a minimum and then increases again to infinity. A transient crack depth  $c_t$  exists for which  $T_c$  is equal for fracture mechanics-based criterion and for stress-based criterion. It can be shown from Fig. 2 that, except at  $c_{min}$ , there are two roots, namely,  $(c_1, c_2)$  for  $c_t$ . If the crack depth is smaller than  $c_1$  or larger than  $c_2$ , the thermal shock resistance  $T_c$  of the plate should be predicted by the stress-based criterion; on the other hand, if the crack depth is between  $c_1$  and  $c_2$ , the thermal shock resistance  $T_c$  of the plate should be predicted by the fracture mechanics-based criterion.

Further consideration of the results reveals that the admissible temperature for thermal shock is less for fracture mechanics-based criterion than for stress-based criterion, at a sufficiently large plate thickness. A transient plate thickness value  $h_t$  has been established exists for which the minimum  $T_c$  is equal for fracture mechanics-based criterion and stress-based criterion. Thus a FGM whose thickness is  $h_t$  has the same thermal shock resistance according to the stress criterion and the fracture mechanics criterion. The fracture mechanics-based criterion is conservative for a plate whose thickness is above  $h_t$ , the stress-based criterion is conservative for a plate whose thickness is below  $h_t$ .

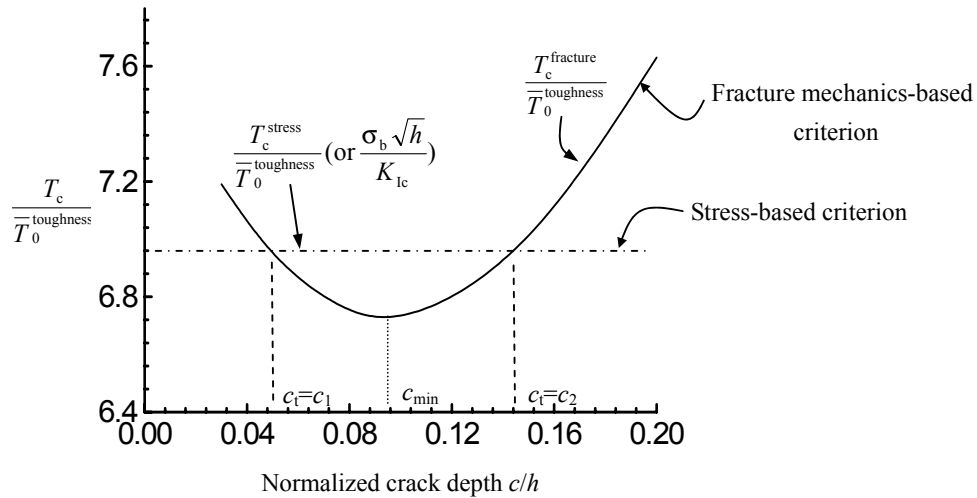


Figure 2. The maximum temperature sustainable by the medium  $T_c$  predicted by the stress-based and by the fracture mechanics-based criteria (Gradient index  $g$  is 2;  $\frac{T_c}{T_0^{\text{toughness}}} = \frac{K_{1c}(\text{ceramic})}{E_c \alpha_c \sqrt{h} / (1-\nu_c)}$ ;  $c$  is the crack depth)

## CONCLUSIONS

The thermal shock behavior of a FGM has been investigated theoretically. The surfaces of the plate undergo a sudden cooling or heating. The temperature and stress histories in the plate are given. The maximum temperature that the material can sustain without catastrophic failure is analyzed according to the maximum local tensile stress criterion and the maximum stress intensity factor criterion. It is found that an FGM with high metal contents exhibits significant resistance to crack growth from ceramic side to metal side. A transient plate thickness value  $h_t$  was obtained for which the admissible temperature is equal for fracture mechanics-based criterion and for stress-based criterion. The thermal shock resistance of a FGM plate with thickness smaller than  $h_t$  will be controlled by the stress-based criterion, and a plate with thickness larger than  $h_t$  will be controlled by the fracture mechanics-based criterion.