

Conservation Laws of Functionally Graded Materials in Elastodynamics

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Keywords: elastodynamics; functionally graded material; Lie's invariance criterion; energy-momentum tensor; conservation law. A special version of Noether's theorem for the sake of absolute invariance on invariant variational principles is applied to the Lagrangian density function for obtaining conservation laws of functionally graded materials. It is found that the mass density and Lamé's coefficients have to satisfy a set of first-order linear partial differential equations. Satisfaction of these equations evolves the infinitesimal symmetry-transformations and the conservation laws in material space. Under the consideration of varying the volume fraction of the constituent materials, the effective mass density and Lamé's coefficients, satisfying those partial differential equations, are obtained. They are numerically well and would not be unrealistic for various FGMs. Four conservation laws in material space are presented. A path-independent integral, which is directly related to the dynamic energy release rate, in the moving coordinate reference attached at the tip of crack is given.

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