NEW FAMILY OF FINITE ELEMENT MODELS FOR COMPOSITE AND NON-UNIFORM POLARIZATION PIEZOELECTRIC STRUCTURES

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Summary The mathematical modelling by FEM of the piezoelectric composite and non-uniform polarization media and ultrasonic transducers are considered. The capabilities of computer program ACELAN for solving piezoelectric problems are described. Continual and finite element formulations of the dynamical problems in compound domains with different physical and mechanical properties (piezoelectric, elastic and acoustic) and semi-discrete FEM approximations and matrix FE systems are given. A set of algorithms, which use the symmetrical saddle matrices to create and solve the FEM equations, are realized in package ACELAN. The one- and two-dimensional piezoelectric finite element and methods of non-uniform polarization modelling are discussed. The new 3D mathematical models of porous and polycrystalline piezocomposites with 3-0 and 3-3 connectivity are proposed.

In presented paper the mathematical modelling by finite element method (FEM) of the piezoelectric devices is considered. Continual formulations of the dynamical problems in compound domains with different physical properties (piezoelectric, elastic and acoustic) are given.
For piezoelectric $\Omega_j = \Omega_{pk}$ and elastic $\Omega_j = \Omega_{ek}$ media we use the following field equations

$$\rho_j \ddot{\mathbf{u}} + \alpha_{dj}\rho_j \dot{\mathbf{u}} - \nabla\cdot\boldsymbol{\sigma} = \mathbf{f}_j; \quad \nabla\cdot\mathbf{D} = 0, \tag{1}$$

$$\boldsymbol{\sigma} = \mathbf{c}_j^E \cdot\cdot(\boldsymbol{\varepsilon} + \beta_{dj}\dot{\boldsymbol{\varepsilon}}) - \mathbf{e}_j^T\cdot\mathbf{E}, \tag{2}$$

$$\mathbf{D} + \zeta_d\dot{\mathbf{D}} = \mathbf{e}_j\cdot\cdot(\boldsymbol{\varepsilon} + \zeta_d\dot{\boldsymbol{\varepsilon}}) + \boldsymbol{\epsilon}_j^S\cdot\mathbf{E}, \tag{3}$$

$$\boldsymbol{\varepsilon} = (\nabla\mathbf{u} + \nabla\mathbf{u}^T)/2; \quad \mathbf{E} = -\nabla\varphi, \tag{4}$$

where α_{dj}, β_{dj}, ζ_d are non-negative damping coefficients, and the other symbols are the standard designations for theory of electroelasticity with the exception of index "j", corresponding for area Ω_j. (For elastic media $\Omega_j = \Omega_{ek}$ the piezomodules $\mathbf{e}_j$ are equal to zero.)
In the equation (1)—(4) we propose the new generalized Kelvin model for damping inputs in piezoelectric analysis [1].
In the case when piezoelectric device is contact with the acoustic media $\Omega_j = \Omega_{al}$ we use the acoustic equation taking into account the linear dissipative effects

$$\frac{1}{\rho_j c_j^2}\dot{p} + \nabla\cdot\mathbf{v}; \quad \mathbf{v} = \nabla\psi, \tag{7}$$

$$\rho_j\dot{\mathbf{v}} = \nabla\cdot\boldsymbol{\sigma}; \quad \boldsymbol{\sigma} = -p\,\mathbf{I} + b_j\nabla\mathbf{v}, \tag{8}$$

where ρ_j is the density; c_j is sound speed; b_j is the damping coefficient for medium $\Omega_j = \Omega_{al}$; p is the sound pressure; $\mathbf{v}$ is the velocity vector; ψ is the velocity potential; $\boldsymbol{\sigma}$ is the stress tensor; $\mathbf{I}$ is the unit tensor.
For solving the problems (1)–(8) we will use FEM in the classical semi-discrete formulation. We approximate the functions $\mathbf{u}$, φ and ψ on the conforming mesh of the finite elements $\Omega_h \subseteq \Omega = \cup_j\Omega_j$ in the form

$$\mathbf{u}(\mathbf{x},t) = \mathbf{N}_u^*(\mathbf{x})\cdot\mathbf{U}(t), \quad \varphi(\mathbf{x},t) = \mathbf{N}_\varphi^*(\mathbf{x})\cdot\boldsymbol{\Phi}(t), \quad \psi(\mathbf{x},t) = \mathbf{N}_\varphi^*(\mathbf{x})\cdot\boldsymbol{\Psi}(t), \tag{9}$$

where $\mathbf{N}_u^*$ is the matrix of the shape functions for displacements, $\mathbf{N}_\varphi^*$ and $\mathbf{N}_\psi^*$ are the vectors of the shape functions for electric and acoustic potentials, $\mathbf{U}(t)$, $\boldsymbol{\Phi}(t)$ and $\boldsymbol{\Phi}(t)$ are the global vectors of nodal displacements, electric and acoustic potentials, respectively.
The standard semi-discrete approximation of FEM generalized settings of the problems (1)–(8) with corresponding boundary and initial conditions leads to the system of differential equations

$$\mathbf{M}\cdot\ddot{\mathbf{a}} + \mathbf{C}\cdot\dot{\mathbf{a}} + \mathbf{K}\cdot\mathbf{a} = \mathbf{F}, \tag{10}$$

$$\mathbf{M} = \begin{pmatrix} \mathbf{M}_{uu} & 0 & \tilde{\mathbf{R}}_{u\psi} \\ 0 & 0 & 0 \\ \tilde{\mathbf{R}}_{u\psi}^T & 0 & -\mathbf{M}_{\psi\psi} \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} \mathbf{C}_{uu} & 0 & \mathbf{R}_{u\psi} \\ \zeta_d\mathbf{K}_{u\varphi}^T & 0 & 0 \\ \mathbf{R}_{u\psi}^T & 0 & -\mathbf{C}_{\psi\psi} \end{pmatrix}, \tag{11}$$

$$\mathbf{K} = \begin{pmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\varphi} & 0 \\ \mathbf{K}_{u\varphi}^T & -\mathbf{K}_{\varphi\varphi} & 0 \\ 0 & 0 & -\mathbf{K}_{\psi\psi} \end{pmatrix}, \quad \mathbf{F} = \left\{ \begin{matrix} \mathbf{F}_u, \\ \mathbf{F}_\varphi + \zeta_d\dot{\mathbf{F}}_\varphi, \\ 0 \end{matrix} \right\}, \tag{12}$$

with unknown vector $\mathbf{a} = \lfloor \mathbf{U}, \mathbf{\Phi}, \mathbf{\Psi} \rfloor^T$. Here $\mathbf{C}_{uu} = \sum_j (\alpha_{dj}\mathbf{M}_{uuj} + \beta_{dj}\mathbf{K}_{uuj})$, where $\mathbf{M}_{uuj}$ and $\mathbf{K}_{uuj}$ are the structural finite element mass and stiffness matrices, and other symbols in (10)—(12) are described in [1, 2].

A set of algorithms, which use the symmetrical saddle matrices to create and solve the FEM equations, is proposed for static and dynamic problems in [3]. Thus the Newmark method without velocities and accelerations node values is used for step-by-step time integration scheme and modified Chollessky decomposition method is used for linear system solver. All procedures that we need in FEM manipulations (the degree of freedom rotations, mechanical and electric boundary condition settings, etc.) are provided also in symmetrical form. FEM for evaluation of natural frequencies of compound electroelastic bodies are investigated. The presented schemes use FEM block matrices, where different matrix blocks are related to different field variables.

On the base of symmetrical algorithms the computer program ACELAN were developed [4–6]. It were tested carefully and the results had been compared with the analogous that were obtained by a well-known computer program ANSYS. The numerical experiments showed that ACELAN and its' algorithms were effective enough and give accurate results.

The algorithms of parallel evaluations for finite element piezoelectric objects are developed for ACELAN [7]. On a designed computational cluster with the use of the new version ACELAN the test calculations of composite elastic and electroelastic complex structures are done.

For one- and two-dimensional piezoelectric structures, such as bars, composite beams and plates, new quasielastic finite elements are obtained [8]. Assembly of these single-type elements include the extrem coupled degree of freedom.

New models for describing process of polarization or repolarization of piezoceramic are also suggested [9]. Mathematical approach for the non-uniform residual polarization in power electric fields is built. It includes incremental theory for the Lagrange approach and calculation schemes of finite element method. Some models of polarization process based on domen's switching physics are supposed. The surfaces of polarization for ideal polarization bodies are constructed.

The different 3D mathematical models of porous and polycrystalline piezocomposites with 3-0 and 3-3 connectivity are built [10, 11]. Finite element programs for the effective constants and effective properties calculations are developed. All the composite properties in the porous and inclusion areas from 0 to 80 % are considered, and some recommendations for more optimal porous piezocomposite building are made. Developed methods are integrated in finite element package ACELAN and ANSYS and are used of the piezoelectric transducers calculations.

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