SHAKEDOWN SAFETY CRITERION IN RELIABILITY ANALYSIS

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Summary This paper discusses the problems related to the integration of the shakedown analysis with the reliability analysis. Failure functions with respect to inadaptation (non-shakedown) are defined in the space of random material/geometrical parameters, load multipliers, and directional cosines. Then, a general problem of time-invariant shakedown reliability analysis is formulated. For special cases with low variance of stochastic parameters, simplified formulations are derived and discussed. Separation of load and resistance terms in the failure functions is suggested. The computation can be further simplified since the shakedown response surface may be computed in advance. The problems may be efficiently solved by reliability simulation methods or FORM/SORM combined with a general shakedown analysis algorithm integrated with the Response Surface Method. A practical industrial example of shakedown reliability evaluations is presented.

INTRODUCTION

Inelastic structures such as, for example, pressure vessels and pipelines subjected to variable repeated/cyclic loading may work in four different regimes: elastic, shakedown (adaptation), inadaptation (non-shakedown), and limit (ultimate) state. Since for the elastic regime there are no plastic effects at all, whereas for the adaptation regime the plastic effects are restricted to the initial loading cycles and then they are followed by exclusively elastic behavior, both regimes are considered as safe working ones and they constitute a foundation for the structural design. Hence the shakedown criterion becomes a significant safety criterion for structural design [1].

The structural shakedown takes place due to the development of permanent residual stresses, which, imposed on actual stresses, shift them towards purely elastic behavior. Residual stresses are a result of cinematically inadmissible plastic strains introduced to the structure by overloads or initial pre-stressing. They may avoid any plastic effects in the future, provided that the loads are smaller than the initial overload. Therefore, a memory effect is observed corresponding to maximum loads.

LOADING DOMAINS

Loading is the major factor influencing the structural reliability. Therefore, its proper modeling needs special attention. Unfortunately, the loading history is usually unknown and only its extreme limits are available. Hence, the shakedown approach, for which the knowledge of exact load history is not necessary, provides a powerful analysis platform. It requires stochastic description of maximum loads (envelopes) over each cycle or extreme limits over the lifetime.

The residual stresses assuring the shakedown may be introduced to the structure by initial plastic deformation or, prior to the exploitation, by some technological processes (e.g. heating, test overload) or pre-stressing. Therefore, it is useful to distinguish the induced (inherently) and imposed (externally) shakedown domains [2]. For reliability evaluation the load and above shakedown domains may be compared providing so called failure conditions. We may distinguish the following approaches:

- **shakedown reliability analysis for the imposed shakedown domain** in which the random load domains are compared with the imposed shakedown domain
- **shakedown reliability analysis for the induced shakedown domain** in which the induced shakedown domains (or maximum load domain) are compared with the theoretical maximum (target) domain to which they may be proportionally enlarged still assuring the shakedown.

SHAKEDOWN RELIABILITY PROBLEMS

Besides the loading, also the material and geometrical parameters should be considered as random variables. As a result, the shakedown multiplier, which separates the safe working regime of shakedown from the inadaptation regimes, becomes a random variable. Then, the general structural shakedown reliability problem may be formulated [3]. The probability of failure (or the corresponding reliability index) against inadapatation (non-shakedown) is defined as the probability that the random load multiplier, defined for the maximum loads, does exceed the limits of the target (or the imposed) shakedown multiplier.

For special cases, with low variance of some stochastic parameters, simplified formulations can be derived from the general shakedown reliability problem. They may be conditioned on vector $\boldsymbol{x}$ of geometric/material parameters, in the case they have a very low variance, as well as conditioned on vector $\boldsymbol{a}$ of dominant direction of extreme load domains. It should be noted that they represent the system reliability and may be obtained in a single analysis without the necessity of considering multiple componental reliability subproblems.

The inadapatation (non-shakedown) failure function has a form, which allows to separate, under certain conditions, the load and the shakedown response terms. This allows for computing the shakedown response functions in advance. Moreover, it is shown that the shakedown response function is equivalent to the failure function. It divides the space of parameters into the safe domain and the failure one. It is recommended to apply the Response Surface Method (RSM) to approximate the response functions [4]. Polynomial and exponential approximations are taken into consideration. A numerical code CYCLONE [5] is integrated to compute the shakedown multipliers required in experiments. For illustration, the shakedown reliability problem is solved for a practical example of high-pressure chamber.

CONCLUSIONS

Since obtained by RSM the shakedown response surface is an analytical one and it constitutes simultaneously the failure function, the reliability computation is significantly simplified and extremely efficient. In particular, the simulation or FORM/SORM methods can be used.

Advantages of the presented method, such as working in the X-space, simple formulation in the low-dimensional load space, accounting for global failure criteria corresponding to the system reliability, separate examination of load and strength parameters as well as introduction of response surfaces are discussed in the conclusions. The methodology presented may be easily applied in the reliability design of realistic structures such as pressure systems.

References

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