RESIDUAL STRESSES IN NONLINEAR STRAIN HARDENING ANNULAR DISKS OF VARIABLE THICKNESS SUBJECT TO EXTERNAL PRESSURE

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Summary A computational model is developed to estimate the residual stresses in nonlinearly hardening variable thickness annular disks under external pressure. The model is based on von Mises' yield criterion, deformation theory of plasticity and a Swift-type hardening law. The residual stresses occurring in annular disks upon complete unloading of external pressure are determined and the possibility of occurrence of secondary plastic flow is investigated.

Theoretical investigation of stresses in rotating or stationary disks subject to different boundary conditions is important for an efficient design and material usage in many industrial applications. Therefore much research has been conducted in this field. Since in general, to better utilize the material, plastic deformation may be admitted to some extent, recent studies focused on elastic-plastic treatment of disks. Residual stresses remain at stand-still of plastically deformed disks and play important role during succeeding loading and unloading cycles. Hence, the knowledge of residual stress distribution is necessary to estimate the strength of the disk.

The objective of this work is to estimate the residual stresses in nonlinearly hardening variable thickness annular disks under external pressure. A computational model based on von Mises' yield criterion, deformation theory of plasticity and Swift hardening law is developed for this purpose. Small deformations and a state of plane stress are presumed. The nonlinear governing differential equation is solved using shooting method combined with Newton iterations. It is assumed that the thickness of the thin annular disk varies continuously in the radial direction r in the form of a general parabolic function $h(r)$:

$$h(r) = h_0 \left[1 - \left(\frac{r}{n+b} \right)^k \right] \qquad (1)$$

where h_0, n and k are parameters ($h_0 > 0$; $0 \le n < 1$; $k > 0$), and b is the outer radius of the disk.

If the inner radius a of the variable thickness annular disk is small, depending on the thickness variation (determined by parameters n and k) plastic region may first develop at the edge or at the inner surface or simultaneously at both surfaces. For $\bar{a} = a/b = 0.2$, the critical geometry at which plastic flow commences simultaneously at both surfaces, are determined to be $n = 0.4$ and $k = 1.02467$. The propagation of elastic-plastic border radius $\bar{r}_{ep} = r_{ep}/b$ with increasing pressures in this nonlinearly hardening disk is shown in Fig. 1.

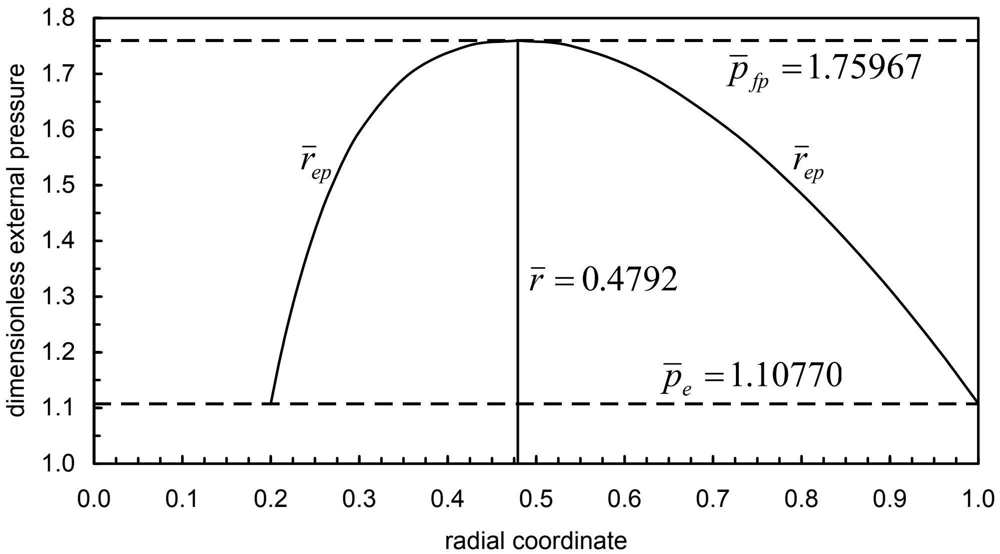

Figure 1. Propagation of elastic-plastic border radius

Plastic deformation begins simultaneously at both surfaces at critical pressure $\overline{p}_e = p_e / \sigma_0 = 1.10770$, with σ_0 being the uniaxial yield limit of the material. Both plastic regions expand with increasing pressures and at $\overline{p}_{fp} = 1.75967$ the disk becomes just fully plastic when the two plastic regions join each other at the radial location $\overline{r} = 0.4792$. For $\overline{p}_e < \overline{p} < \overline{p}_{fp}$ disk consists of three regions: inner plastic, elastic and outer plastic.

The dimensionless residual stresses upon removal of elastic-plastic pressure $\overline{p} = 1.7 > \overline{p}_e$ are computed and plotted in Fig. 2. The stress variable ϕ^R in this figure is computed in terms of residual stresses from $\phi^R = [(\sigma_r^0)^2 - \sigma_r^0 \sigma_\theta^0 + (\sigma_\theta^0)^2]^{1/2}$. With this definition, according to von Mises' yield criterion, secondary plastic flow occurs when $\phi^R \geq 1$. Fig. 3 shows the residual stresses upon complete removal of fully plastic limit pressure $\overline{p}_{fp} = 1.75967$. It is evident from this figure that for the disk with parameters $\overline{a} = 0.2$, $n = 0.4$ and $k = 1.02467$, secondary plastic flow does not take place during all elastic-plastic deformation stages.

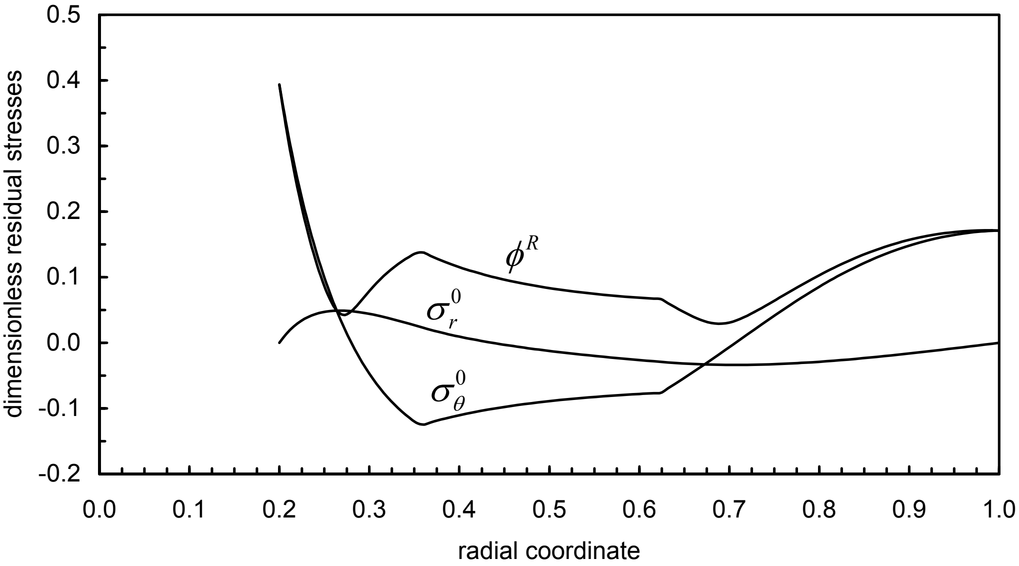

Figure 2. Residual stresses upon removal of elastic-plastic pressure $\overline{p} = 1.7$

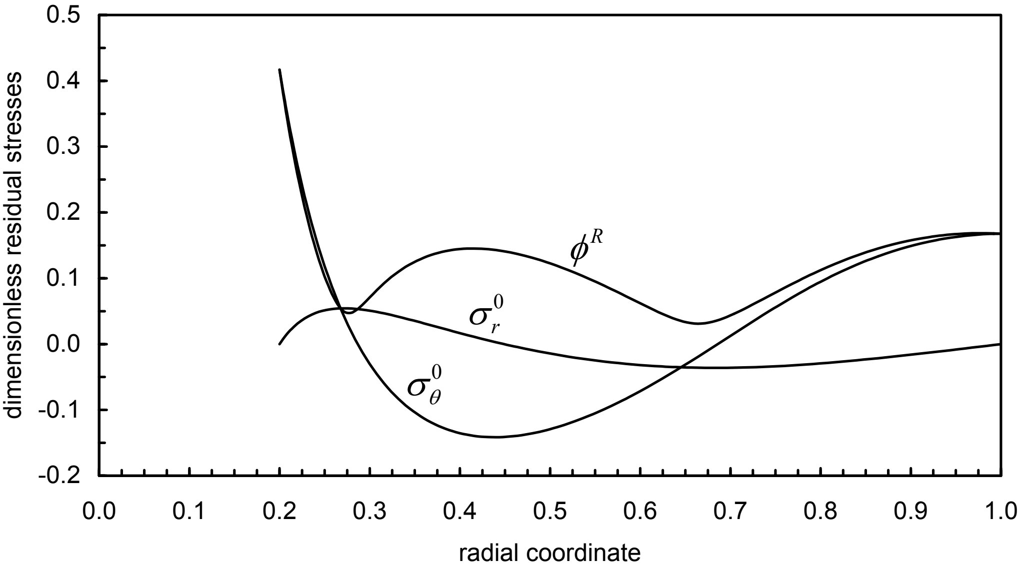

Figure 3. Residual stresses upon removal of plastic limit pressure $\overline{p}_{fp} = 1.75967$