MECHANICAL FEATURES OF PIANO HAMMER FELT

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Summary Experimental testing of piano hammers, which consist of a wood core covered with several layers of compressed wool felt demonstrates, that all hammers have the hysteretic type of the force-compression characteristics. It is shown, that different mathematical hysteretic models can describe the dynamic behavior of the hammer felt. In addition to the four-parameter nonlinear hysteretic felt model, another new three-parameter hysteretic model was developed. The both models are based on the assumption that the hammer felt made of wool is a microstructured material possessing history-dependent properties. Both of the models are equivalent for the slow loading of the felt.

INTRODUCTION

The dynamics of the hammer - string interaction is one aspect of piano physics that has been the subject of considerable research, beginning with von Helmholtz. The hammers in the early pianos have been made of wooden head covered with leather. Since about 1830, the standard material for piano hammers manufacturing is the felt. In spite of the endless attempts to match a more suitable material for the piano hammer, the felt is a unique coating matter of wooden mallets used up to the present. The modern hammers have a wood core covered with one or two layers of compressed wool felt, whose stiffness increases from heavy bass hammers to light treble hammers to produce a good tone.

First constitutive framework proposed to mathematical model of the hammer felt was made by Ghosh [1], who considered the force-compression characteristic of the felt obeying the power law form

$$F = K\,u^p\,, \tag{1}$$

where F is the acting force, u is felt compression, and constant K has units of N/m^p. Experimental static testing of different hammers by Hall and Askenfelt [2] demonstrate that for hammers taken from pianos the values of p ranging from 2.2 to 3.5 give a good approximation of dependence (1).

SIMPLE HAMMER FELT MODEL

A new geometrical version of the static hammer felt model is proposed in [3]. By deriving of this model it was taking into account that the felt deformation depends only on the stiffness of the felt material, identical for all the hammers of the piano, and on the geometrical configuration of the felt compressed. Assuming that the kinetic energy of the hammer transforms into the energy of the felt deformation and that the energy density is a constant value in the volume of interaction, the simple model of the felt is derived in the form

$$F(y) = E\frac{d^3}{R}\left(1+\frac{d}{2R}\right)^{-1/2} y\,q(y)\,, \tag{2}$$

with a function

$$q(y) = 2\arcsin\phi - 2(1-2y)\phi\,; \qquad \phi = 2\sqrt{y(1-y)}\,.$$

Here $y = u/d$ is a nondimensional compression; E is a Young's modulus of the felt material; d is the string diameter; R is the curvature radius of the hammer head. For small deformation $y \ll 1$, Eq. (2) gives

$$F(u) = \frac{\sqrt{2}E}{R}\left(\frac{rR}{r+R}\right)^{1/2} u^{5/2}\,, \tag{3}$$

which is close analogy to the Hertz's law. The value of the compliance nonlinearity exponent $p = 2.5$ in Eq. (3) is in a good agreement with the experimental results discussed in [2]. At the same time, according to Hertz's Law the force acting on two connected locally Hookean bodies gives p=1.5. The values of p different from 1.5 indicate the non-Hooke or the nonlocal felt properties. Just like these properties of the felt were confirmed experimentally by Yanagisawa, Nakamura and Aiko [4], and by Yanagisawa and Nakamura [5]. Their dynamic experiments demonstrate very important properties of the felt: the nonlinear force-compression characteristic, strong dependence of the slope of the loading curve on the rate of loading, and significant influence of hysteresis, i.e. the loading and unloading of the felt are not alike. These phenomena require that the felt made of wool is a microstructural material possessing history-dependent properties. The dynamic behavior of such solid matter is highly sensitive to characteristic frequency and rate of loading, and for this reason the concept of an almost unique force-compression curve for a given material does not exist. The model that describes the history-dependent properties of the hammer felt was proposed in [6].

HYSTERETIC MODELS OF THE FELT

The new hysteretic model is derived by replacing of the constant parameter K in expression (1) by a time-dependent operator $F_0\ [1 - R(t)*]$, where $*$ denotes the convolution, and the relaxation function given by $R(t) = (\varepsilon/\tau_0)\exp(-t/\tau_0)$. Thus, instead of the simple relation (1) we have the four-parameter hereditary model of the felt in the form

$$F(u(t)) = F_0 \left[u^p(t) - \frac{\varepsilon}{\tau_0} \int_0^t u^p(\xi) \exp\left(\frac{\xi - t}{\tau_0} \right) d\xi \right] . \tag{4}$$

Here the instantaneous stiffness F_0 and compliance nonlinearity exponent p are the elastic parameters of the felt, and hysteresis amplitude ε and relaxation time τ_0 are the hereditary parameters.

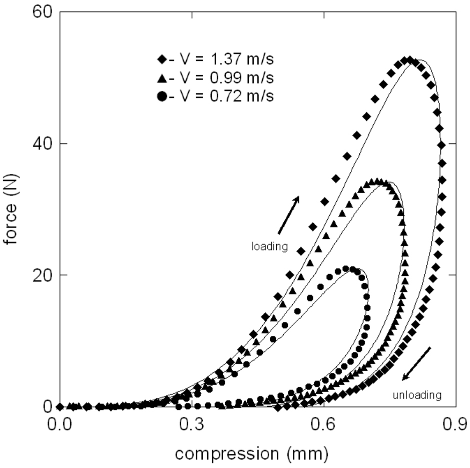

Figure 1. Force-compression characteristics of the felt. Experimental data and numerical simulation.

According to this model, a real piano hammer felt possesses history-dependent properties, or in other words, is a material with memory. For the experimental study of the dynamic felt features the piano hammer testing device described in [7] was used. This device permits to measure the force-time and compression-time dependencies, and investigate the force-compression characteristics of the felt under the different rates of loading. The results of the experimental felt examining are presented in Fig. 1. The arrows indicate the direction of the compression process. The solid lines here represent the numerical simulation of the experiment in according to hysteretic model (4).

The relationships of dynamic force versus felt compression show the significant influence of hysteresis characteristics, so the loading and unloading of the felt (shown by arrows) are not alike. Moreover, the slope of the force-compression characteristics increases with the growth of the hammer velocity, just like the model of the hysteretic hammer predicts. It is evident, that the experiments confirm fairly well the theory. It is also evident that in according to this model each force–compression curve results in a unique combination of felt parameters and vice versa.

However not all is so simple. In spite of this almost evident supposition, the numerical simulation of the felt impact demonstrates that very similar (by eye) force-compression curves can be obtained using the different sets of felt parameters. A close and subtle analysis of this phenomenon results in a new and quite another hysteretic model of the felt.

Usually the hammer velocity not exceeds 5 m/s. This fact corresponds to the non equality $F(t) \gg \tau_0\ dF/dt$, which is valid for all values of τ_0 that are rather small in comparison with the contact duration. Taking into account this non equality and eliminating the integral term in Eq. (4) we can determine the new three-parameter model of the felt in the form

$$Q(u(t)) = Q_0 \left[u^p + \alpha \frac{d(u^p)}{dt} \right] , \tag{5}$$

where $Q(u)$ is the acting force, Q_0 is the static felt stiffness, and α is the retarded time. Similar to model (4), this model of the felt can also be simply proposed by replacing of the constant parameter K in expression (1) by a time-dependent operator $Q_0\ [1 + \alpha\, D]$, where D denotes time differentiation.

Such hysteretic model (5) is very similar to nonlinear Voigt model and permits a description of the felt compression that is consistent also with experiments. Both of the models describe the dynamic felt behavior in a similar way up to the rates of loading 10 m/c. In case of very slow deformation, the both models are complete equivalent. For very fast loading these two models are rather different. Using only the general consideration it is very difficult to choose which model is more physical and reasonable by nature. To decide this problem and to prefer the correct model the additional experiments with a very fast felt loading must be provided.

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