WAVE—MEAN FLOW INTERACTION IN COUPLED ATMOSPHERE–ICE–OCEAN SYSTEMS

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Summary A dynamically consistent framework for modelling atmosphere–ocean interaction must take account of surface waves, either implicitly or explicitly. We may account for the waves explicitly by employing a numerical spectral wave model, and applying a suitable theory of wave—mean flow interaction. Below the water surface the generalized Lagrangian mean (GLM) formulation is suitable, and a closed system of equations may be obtained to second order in wave slope by applying wave action conservation equations in the propagation of the spectral wave components. The coupled model system will also take account of the Earth's rotation, the momentum balance during wave generation and dissipation, the effect of depth-varying currents on wave propagation, the presence of surface films and sea ice, the effect of waves on the mean water level, and the generation of Langmuir circulations.

INTRODUCTION

Surface waves comprise an important aspect of the interaction of the atmosphere and the ocean. Any dynamically consistent framework for the modelling of atmosphere–ocean interaction must take them into account, either implicitly, via a Charnock-type dependence of aerodynamic surface roughness on the air–sea momentum flux or wind speed, or explicitly, using, for example, a spectral wave prediction model. In the latter case, the momentum flux from the airflow to the waves may be computed by analytical or numerical models for wave generation, or by related quasilinear theory [1, 2, 3]. The flux of momentum from the waves to the ocean current may be a result of wave energy dissipation, by breaking [4], or by turbulent stresses within the water column [5, 6] or at the sea bottom [7]. A third method of representation of the dynamical effect of waves is by inertial coupling theory [8, 9], in which the wave field acts to transport momentum vertically, from the critical level in the atmospheric boundary layer, where the dominant wave phase speed is equal to the wind speed, to a level below the wave troughs where the current approximates the wave-induced Stokes drift.

COORDINATE SYSTEMS

In a spectral wave model, such as, for example, WAM [10], we compute the distribution of the variance of the surface displacement amongst components of different wavenumber, and so may couple such a model with an ocean current model by means of an $O(\epsilon^2)$ interaction theory where ϵ is a representative wave slope. Although we may formulate the problem in fixed Cartesian (Eulerian) coordinates, there are advantages to using curvilinear coordinate systems which follow the water surface during the course of a wave cycle. We may resolve vertical variations in the near-surface mean flow and other variables, at small distances from the surface, which can be essential for applications such as heat and gas exchange through the water surface, and ice formation.

Although it is possible to employ a curvilinear coordinate system in which the coordinate displacements are purely vertical [11], it may be more satisfactory to allow the coordinate surfaces also to move horizontally. A particularly useful coordinate system is employed in the generalized Lagrangian mean (GLM) formulation of Andrews and McIntyre [12], in which the mean velocity $\overline{\mathbf{u}}^L$ and the coordinate displacement ξ are related by $\partial\xi/\partial t + \overline{\mathbf{u}}^L \cdot \nabla\xi = \mathbf{u}^\xi - \overline{\mathbf{u}}^L$, where $\mathbf{u}^\xi$ is the instantaneous velocity in the curvilinear coordinate frame. The coordinate system is close to a Lagrangian coordinate system for small values of $\overline{\mathbf{u}}^L$, so we may use results from Lagrangian perturbation expansions.

The effect of the GLM mean flow on wave propagation can also be formulated in terms of finite-amplitude conservation laws for wave action [13], so that to $O(\epsilon^2)$, a closed system of equations for the coupled wave—mean flow system may be obtained. A systematic treatment of wave–current interaction in two dimensions (horizontal and vertical) was given by Groeneweg and Klopman [14]. The GLM formulation cannot be used in the vicinity of critical layers, so more general curvilinear coordinates are required for airflow over waves, cf. Jenkins [3].

OUTLINE OF A COUPLED SYSTEM

For a coupled wave–mean flow model system in a region possibly covered by sea ice, we require the following factors to be included:

1. *The Earth's rotation* is included in the GLM equations [12]. We may, neglect terms of higher order than $O(f/\sigma)$, where f is the Coriolis parameter and σ is the wave intrinsic angular frequency. The effect of Coriolis force and vorticity in the mean current on the Stokes drift also comes naturally from the GLM equations.

2. Momentum balance in wave generation. This comes from the surface oscillatory boundary conditions for pressure and shear stress, see [3, 5, 6].

3. *Momentum balance in wave dissipation*. Wave pseudomomentum which is lost through dissipation will reappear as a forcing of the mean current.

- *Wave breaking* may be nevertheless often regarded as 'weak in the mean' [16], and we may simulate it empirically as dissipation by other means (e.g. turbulence, viscosity).
- *Within the water column*. Dissipation of waves by turbulence (or by breaking, see above) may be simulated by an eddy viscosity which, however, in practice must be less than that used to diffuse momentum within the mean flow field [5, 6, 17]. If the eddy viscosity varies with the vertical coordinate, there is a source of mean momentum within the water column [5, 6].
- *Surface and bottom boundary layers*. Analytical solutions are most straightforward if we assume the eddy viscosity ν_E to be constant within the boundary layers. The surface boundary layer modifies the boundary conditions for the fluid in the interior. The no-slip bottom boundary condition results in a relatively strong mean flow [7].
- *Sea ice and surface films* alter the boundary conditions radically, increase the wave damping substantially, induce a strong near-surface mean flow from the dissipation of wave pseudomomentum, and may lead to convergence and downwelling at the ice edge [18, 19, 21].

4. *Effect of the mean flow on wave propagation*. Waves on a vertically-sheared current obey the Orr–Sommerfeld (or Rayleigh) equation. If the shear in the current is weak, the wave phase speed is changed by the weighted integral of the current, where the weighting factor is e^{2kz} in deep water, k being the wavenumber and z the (upward) vertical coordinate [22].

5. *Changes in sea-surface elevation*. One disadvantage of the GLM equations is that the Jacobian determinant of the coordinate transformation is not constant, which makes the continuity equation more complex than its counterpart in Eulerian coordinates. If this effect is properly taken into account, the 'set down' of the mean sea surface is reproduced, as waves propagate without breaking into shallow water, as well as the wave setup which occurs as they dissipate [23, 24].

6. *Langmuir circulations*. The Craik–Leibovich equations [25] are straightforward to derive from the GLM equations [26]. The helical roll circulations which are solutions to these equations, either via an instability mechanism, or as a result of forcing by crossing wave trains [27], may either be simulated directly by the coupled model equations, or averaged out if a grid scale coarser than the circulation scale is used.

The Coriolis and dissipative effects mentioned above can be regarded as 'small' perturbations, so may be added together in the equations for the mean flow.

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