

Spontaneous Generation of Inertia-gravity Waves by Balanced Motion

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The spontaneous generation of inertia-gravity waves by balanced motion leads to fundamental limitations in the accuracy of balanced models. In the standard quasi-geostrophic regime (with small Rossby number and order-one Burger number) the amplitude of the inertia-gravity waves can be expected to be exponentially small in the Rossby number. We demonstrate this explicitly by deriving asymptotic estimates for this amplitude in two models described by ordinary differential equations: the five-component model of Lorenz and Krishnamurthy; and a model describing the evolution of sheared disturbances in a three-dimensional Boussinesq fluid. In both cases the asymptotic estimates are confirmed by numerical experiments. The concepts and techniques used in our analysis (optimal truncation of asymptotic series, Borel–Laplace transform, complex-time dynamics) help clarify a number of issues in balanced dynamics and initialization. Their relevance to more realistic models of the atmosphere and oceans is discussed.

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