TRANSITION THRESHOLDS IN MICROCHANNELS UNDER THE EDL EFFECT

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We have recently shown that the electric double layer destabilizes considerably the micro-channel flows (Tardu, 2003 a and b). It is recalled that the velocity profile under the electrokinetic EDL effect can be put in non-dimensional form as:

$$u = 1 - y^2 - 4\frac{I_1 - I_2}{\frac{\kappa^2 \sinh\kappa}{\bar{\zeta}^2 G} + 4\left(I_3 - \frac{I_4}{\sinh\kappa}\right)}\left\{1 - \left|\frac{\sinh\kappa y}{\sinh\kappa}\right|\right\} \quad (1)$$

where the scaling velocity is the centerline velocity of the Poiseuille component, i.e. $-\frac{a^2\, dp/dx}{2\mu}$ and the scaling length is the half channel height a. There are several parameters in this equation, for instance $G = \frac{(n_0 z e a)^2}{\lambda_0 \mu}$, with n_0 standing for the ionic number concentration, z for the valence of positive or negative ions, e for the electron charge λ_0 the electric conductivity of the fluid, and μ for its dynamic viscosity. One of the most important quantities involving in (1) is the non dimensional Debye-Hückle parameter $\kappa = ak = a\left(2 n_0 z^2 e^2 / \varepsilon\varepsilon_0 k_b T\right)^{1/2}$ with ε and ε_0 being respectively the dielectric constant of the medium and the permittivity of vacuum, k_b the Boltzmann constant and T the absolute temperature. The characteristic EDL thickness is $1/k$. The non-dimensional Zeta potential reads for $\bar{\zeta} = \frac{ze\zeta}{k_b T}$. The quantities I in (1) are given by:

$$I_1 = I_3 = \frac{\cosh\kappa - 1}{\kappa}, I_2 = \left(\frac{1}{\kappa} + \frac{2}{\kappa^3}\right)\cosh\kappa - \frac{2}{\kappa^2}\sinh\kappa - \frac{2}{\kappa^3}, I_4 = \frac{\sinh\kappa\cosh\kappa}{2\kappa} - \frac{1}{2} \quad (2)$$

The important difference between the EDL and macro-Poiseuille flow profiles is the presence of an inflexional point at $y \approx \frac{1}{\kappa}\operatorname{arcsin}h\left\{-\frac{2}{r\kappa^2}\sinh(\kappa)\right\}$ in the EDL profile where r is the ratio of the EDL and Poiseuille flows centerline velocities. This makes the flow inviscidly unstable, according to the Fjortoft's criteria. The neutral curves deduced from the hydrodynamic stability analysis are summarized in Fig.1. It is clearly seen that the critical Reynolds number decreases by a factor nearly equal to 2 under the EDL effect at $\kappa = 41$: the critical wave and Reynolds numbers of the microflow are respectively $\alpha_c = 1.10$ and $Re_c = 3190$, to be compared with $\alpha_c = 1.02$ and $Re_c = 5772$ of the conventional Poiseuille flow. This effect can be more appreciated if it is recalled that, at $\kappa = 41$, the friction factor increases by only some 10%. It is clear that one of the most significant effects of EDL is the decrease of the critical Reynolds number, rather than the increase in friction coefficient or the apparent viscosity.

The non linear saturation of the primary stability and formation of a secondary flow, together with the secondary instability processes have to be analyzed in EDL flow similarly to the Poiseuille macro-flow .Some arguments on the reinforcing effect of the EDL on the subcritical nature of the macro Poiseuille flow may however already be given. The square of the amplitude of a finite disturbance is given by:

$$\frac{d\left|A_1^2\right|}{dt}=2\alpha c_i\left|A_1\right|^2+\left(k_1+k_2+k_3\right)\left|A_1\right|^4 \quad (3)$$

according to Stuart [13]. The flow reaches a subcritical equilibrium state when $k_1+k_2+k_3>0$. The coefficient k_1 represents the distortion of the mean motion: it is related to the eigenfunctions of the linear stability problem, and it is negative. The coefficient k_2 is linked to the generation of the harmonic of the fundamental and is also likely negative. The wall normal distortion of the fundamental (k_3) must " be positive and outweigh the combined negative effect of k_1 and k_2 to reach a subcritical state". Now, k_1 is proportional to $\mathrm{Re}_c\,\alpha_c^2$. It has therefore a significantly smaller negative contribution to $k_1+k_2+k_3$ under the EDL effect. Furthermore, part of the terms involving in the coefficient k_3 is inversely proportional to Re_c and the EDL presumably reinforces the positive character of k_3 in the subcritical state.

The analyze of the EDL effect on the nonlinear stability mechanism is performed through the ***spatio-temporal development of a spot in a channel flow by Direct Numerical Simulations*** in this investigation. A perturbation related to a pair of counter rotating vortices is followed in time and space with and without EDL. Results show the profound destabilizing EDL effect providing that the liquid contains a very small amount of ions with large enough Zeta potential and low conductivity/viscosity.

References

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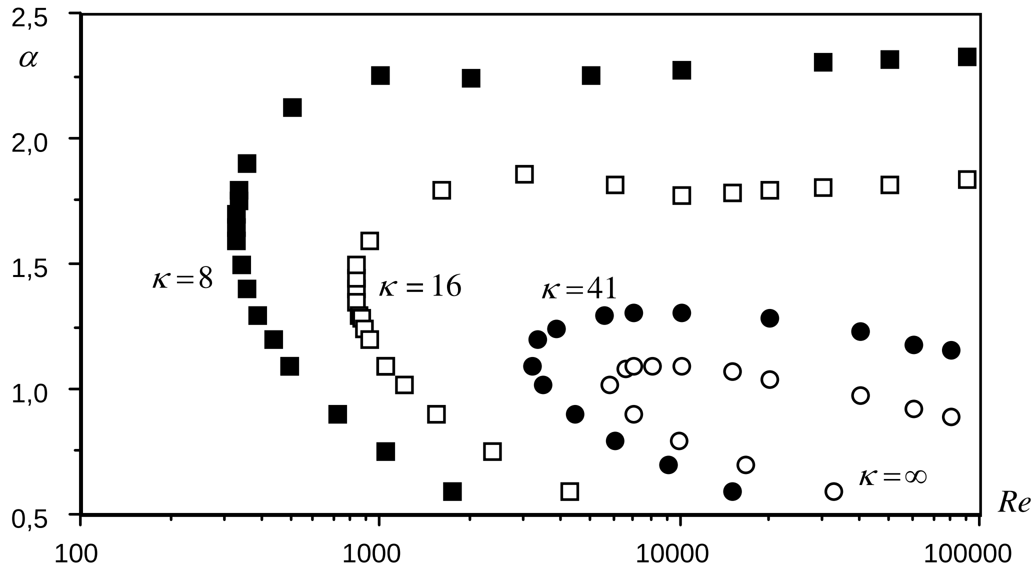

Figure 1 Neutral curves of the EDL flow compared with the Poiseuille flow. a) The open circles correspond to Poiseuille flow with $\kappa=\infty$. Bold circles correspond to $\kappa=41, G=12720$ and $\bar{\zeta}=2.1254$. The rest of the results are obtained by changing the microchannel height and keeping constant the rest of the parameters. The triangle is obtained for $\kappa=164$ b) Neutral curves for κ=8, 16 and 41 compared with the macroscale flow.