Wrinkling instability in nanolayers; anisotropy and sliding effects

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Abstract

Following an idea of Suo *et al.* [1], we extend the linear stability analysis of a two-layer structure, intended to model the relaxation of a thin elastic film on a viscous layer, to account for both anisotropy of the elastic film and friction at the interface between the film and the substrate. The main application concerns the feasibility of strain relaxed InAsP and InGaAs compliant substrates. We compare our theoretical estimates for both the orientation and the wavelength of the periodic undulation of the film with experimental results obtained for $In_{0.65}Ga_{0.35}As$ on a Si host via Apiezon wax.

1 Formulation

The thin elastic film of thickness h is modelled using an anisotropic version of the von-Kàrmàn theory; let C_{11}, C_{12} and C_{44} denote the classical elasticites for a cubic material, and define $Q_{11} = C_{11} - C_{12}^2/C_{11}$, $Q_{12} = C_{12} - C_{12}^2/C_{11}$ and an anisotropy parameter $\xi = (C_{44} - C_{12})/C_{11} - 1$. We use $(\boldsymbol{u}, w)$ for the displacement field in the film and recall that the anisotropic version of the von-Kàrmàn theory provides the equilibrium equations in the form

$$p = \frac{h^3}{12}\left[Q_{11}\left(\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial y^4}\right) + 2(Q_{12} + C_{44})\frac{\partial^4 w}{\partial x^2 \partial y^2}\right] - \boldsymbol{T}\cdot\nabla w - \boldsymbol{N} : \nabla(\nabla w), \quad (1)$$

$$\boldsymbol{T} = \nabla\cdot\boldsymbol{N}, \quad (2)$$

where $\boldsymbol{N} = \boldsymbol{C}[\boldsymbol{\varepsilon}_0 + \boldsymbol{\varepsilon}(\boldsymbol{u}) + \boldsymbol{f}(\nabla w)]$, $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{x}\otimes\boldsymbol{x}/2$, $\boldsymbol{\varepsilon}(\boldsymbol{u})$ is the classical small-strain tensor, $\boldsymbol{C}[\boldsymbol{a}] = h\,[Q_{12}\mathrm{tr}(\boldsymbol{a})\boldsymbol{I} + (Q_{11} - Q_{12})\boldsymbol{a}]$ and $\boldsymbol{\varepsilon}_0 = \varepsilon_0\boldsymbol{I}$ denotes the misfit.

In the full three-dimensional situation, following Suo *et al.* [1], we use the lubrication approximation for the isotropic incompressible viscous layer. If H denotes the thickness of the viscous layer and η the viscosity, the velocity field at the interface between the viscous layer and the elastic film is given by

$$\boldsymbol{v} = -\frac{H^2}{2\eta}\nabla p + \frac{H}{\eta}\boldsymbol{T}, \quad (3)$$

$$\dot{w} = \frac{H^3}{3\eta}\Delta p - \frac{H^2}{2\eta}(\nabla\cdot\boldsymbol{T}), \quad (4)$$

when the continuity of the displacement (thus of the velocity field) and continuity of $(\boldsymbol{T}, p)$ at the interface between the viscous layer and the elastic film are assumed. Using (1) and (2) in (3) and (4) we obtain a nonlinear system of partial differential equations for $(\boldsymbol{u}, w)$. A trivial solution is $(\boldsymbol{u}, w) = (\boldsymbol{0}, 0)$ that corresponds to a homogeneous prestressed flat film.

2 Linear stability analysis

We perform a linear stability analysis near the trivial solution looking for the growth or decay of a perturbation having the general form

$$(\boldsymbol{u}(\boldsymbol{x}, t), w(\boldsymbol{x}, t)) = (\boldsymbol{u}_0(t), iw_0(t))e^{i\boldsymbol{k}\cdot\boldsymbol{x}}. \quad (5)$$

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Using (5), after linearization, a straightforward method leads to a linear system of ordinary differential equations in the form

$$\frac{d}{dt}\begin{pmatrix} \boldsymbol{u}_0 \\ w_0 \end{pmatrix} = \frac{Hh}{2\eta}\boldsymbol{A}\begin{pmatrix} \boldsymbol{u}_0 \\ w_0 \end{pmatrix} \tag{6}$$

for a constant matrix $\boldsymbol{A}$.

Necessary and sufficient conditions for the decay of perturbations are given by

$$\mathrm{tr}\boldsymbol{A} < 0, \qquad \det\boldsymbol{A} > 0, \qquad \det\boldsymbol{A} - (\mathrm{tr}\boldsymbol{A})((\mathrm{tr}\boldsymbol{A})^2 - \mathrm{tr}\boldsymbol{A}^2)/2 > 0. \tag{7}$$

The analysis of (7) provides the following results:

(a) An instability appears only if $\varepsilon_0 < 0$, i.e., in compression.

(b) Short wavelengths perturbations are unstable while long wavelength perturbations are stable; these results recover previously results obtained by Suo et. al. [1]

(c) As expected, the orientation of the maximum growth velocity perturbation depends on the anisotropy parameter ξ; the analysis shows that if $\xi > 0$ the instability manifests in the [100] direction while if $\xi < 0$ the preferred direction is [110].

(d) We also analysed a more general situation involving interfacial conditions that account for discontinuous tangential velocity field at the interface. Using an interfacial law of the form

$$\dot{\boldsymbol{u}}^{\mathrm{film}} - \boldsymbol{v}^{\mathrm{fluid}} = \zeta \boldsymbol{T}^{\mathrm{fluid}} \tag{8}$$

for some $\zeta > 0$, we prove that both the critical wavelength and the maximum velocity wavelength decrease.

3 Experimental evidence

We compare our theoretical results with experimental evidence obtained in [2]; for $\mathrm{In}_{0.65}\mathrm{Ga}_{0.35}\mathrm{As}$ on Apiezon wax the AFM microscopy images show periodical undulation of the film in the [100] direction, with a wavelength $\lambda_{\mathrm{max.}}^{\mathrm{exp.}} \sim 4.8\ \mu$m. Numerical computations using our results show that for $\mathrm{In}_{0.65}\mathrm{As}_{0.35}\mathrm{As}$ the anisotropy parameter equals $0.47 > 0$, thus predicting the experimentally observed orientation. The maximum growth rate is attained for

$$\lambda_{\mathrm{max.}} = 2\pi h \sqrt{\frac{3(C_{11}^2 - C_{12}^2)}{-12\varepsilon_0(C_{11}^2 + C_{11}C_{12} - 2C_{12}^2)}}. \tag{9}$$

In our situation $h = 30$ nm, $\varepsilon_0 = -0.085$ and we find $\lambda_{\mathrm{max.}}^{\mathrm{th.}} = 0.7\ \mu$m. This difference between the predicted wavelength and the experimental evidence may be the result of a partial sliding at the interface between the film and the viscous substrate. This result was further improved using an interfacial condition as in (8), that leads to a decrease of the theoretical wavelength as mentioned above in (d).

References

[1] R. Huang, Z. Suo, Wrinkling of a compressed elastic film on a viscous layer, *Journal of Applied Physics*, **91**, 1135-1142, 2002.

[2] M. Kostrzewa et. al., Feasibility of strain relaxed InAsP and InGaAs compliant substrates, *Ph.D Thesis*, Ecole Centrale de Lyon, 2003.