INSTABILITIES OF COMPOSITE MATERIALS REINFORCED BY NANO-FIBRES : A RE-EXAMINATION OF ELASTIC BUCKLING

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Elastic buckling is studied as a possible failure mechanism of a composite. Although it appears that common polymer matrix composites (e.g. glass/epoxy or carbon-graphite/epoxy) undergo plastic micro-buckling or fail due to other mechanisms (such as fibre crushing, delamination, etc.)[Fleck (1997)], nevertheless, for very dilute composites fabricated in the emerging framework of nanotechnology, elastic buckling may, in fact, be the dominant failure mechanism.

The phenomenon of elastic buckling of composites was first investigated in the 1960's. Since then, it has been generally accepted that elastic buckling of a composite consisting of straight fibres in a matrix can occur in two modes : a shear buckling mode (SBM), where the fibre and matrix exhibit in-phase deformation (i.e. assume the same deformed shape and a transverse buckling mode (TBM) where the fibre and matrix exhibit anti-phase deformation Moreover, it is asserted that transverse buckling occurs for relatively low values of the fibre volume fraction, v_f, while shear buckling takes place for larger values of v_f [Jones (1975)]. These two types of buckling modes were first studied by Rosen (1964) using, in both cases, simple approximations together with an energy approach.

For the case of shear buckling, assuming in-plane buckling, Rosen treated the problem by means of a model consisting of thin layers embedded in a matrix and thus obtained a buckling stress [see Rosen (1964) and Jones (1975)]

$$\sigma = R_0 + R_1 \left(\frac{H_f}{L}\right)^2, \qquad (1.1b)$$

where

$$R_0 = \frac{G_m}{v_m}, \qquad R_1 = \frac{\pi^2}{3} E_f\, v_f. \qquad (1.1c, d)$$

In the above, G_m denotes the shear modulus of the matrix material, E_f Young's modulus of the fibre, and $v_m = 1 - v_f$, the volume fraction of the matrix; H_f is the fibre (layer) thickness and L is the buckling wavelength (which is not specified in the analysis). Clearly, according to this solution, the lowest buckling mode occurs as the wave length $L \to \infty$ with a corresponding buckling stress,

$$\sigma_R^{(S)} = \frac{G_m}{v_m}.$$

In his investigation of the transverse buckling mode (TBM), Rosen, derived an expression for the critical buckling stress,

$$\sigma_R^{(T)} = 2v_f \left(\frac{E_f E_m R_v}{3}\right)^{1/2}$$

where E_m denotes the Young's modulus of the matrix and $R_v = \frac{v_f}{v_m}$ denotes the ratio of the volume fractions.

The Rosen solutions have, since then, been widely used and have been applied to common composites such as glass/epoxy. For these classes of composites with stiffness ratios E_f/E_m of the order of 20, the Rosen solution in the shear buckling mode leads to buckling stresses with corresponding strains – clearly unrealistic – of the order of 10% and higher. Indeed, Rosen and Dow (1972) pointed out the unreasonableness of such strains. Nevertheless, to the authors' best knowledge, in all existing literature devoted to buckling of composites, the Rosen approximation is adopted.

In the present investigation, assuming initially the existence of the two possible modes of buckling, we consider the problem of elastic buckling of a composite consisting of periodically spaced layers, using as a model, fibres embedded in an elastic matrix for which an exact 2-D stress fields is determined. By investigating the two modes of elastic buckling, we show that the stress required to cause buckling in the TBM is always greater than that for the SBM. For the particular case of dilute composites, corresponding to nano-fibre reinforced composites, the TBM stress is seen to converge (from above) to the SBM stress. Thus we demonstrate that the transverse buckling mode is, in fact, a spurious mode and that buckling of a composite always occurs in the SBM. In addition, we show that while buckling of non-dilute composites in the SBM occurs with infinite wavelengths, for relatively dilute composites, buckling in this mode occurs with *finite* wavelengths and therefore the approximate Rosen solution fails to be valid in these cases.

Since the investigation under consideration is concerned with buckling of composites as an instability phenomenon, a linear analysis is used here. In fact, as opposed to the Rosen solution, in the present investigation, elastic buckling is found to occur with realistic strains within the elastic range of composites having relatively small fibre volume concentrations and high stiffness ratios, E_f/E_m. Such cases may be found, for example, in composites reinforced by carbon nano-fibres of high stiffness.

Indeed, carbon nanotubes, whose estimated values of Young's modulus as reported by Treacy, Ebbensen and Gibson (1996) range up to 4.15 TPa (with an average value of 1.8 TPa), appear to show great promise as fibres in light-weight composites. Based on recent experiments, Wagner et al (1998) give estimates of the maximum tensile stress of such fibres (when embedded in a polymer matrix) of approximately 55 GPa. Moreover, based on experimental investigations of buckling of multi-wall embedded carbon nanotubes of finite length, Lourie, Cox and Wagner (1998) calculated, on the basis of simple shell theory, critical stresses of 135 GPa to 147 GPa. While common epoxy materials (e.g. BP907 and AF126) can undergo strains with linear elastic behavior of the order of 2% simulated experiments of individual isolated nanotubes show that, due to their high strength and morphology, they can exhibit strains greater than 10% in tension and 5% in compression [Yakobson et al. (1996)]. A comprehensive review of nanofibre technology is presented in Yakobson and Smalley (1997).

As a result of these developments, a re-examination of elastic buckling as a realistic failure mechanism in the framework of nanotechnology appears warranted.

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