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NUCLEATION OF CRACKS IN TWO-DIMENSIONAL PERIODIC CELLULAR MATERIAL

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Summary The brittle fracture behavior of cellular material modeled as 2D regular beam lattice is examined. The flaw development from a single failure of a beam to a macrocrack produced by multiple beam breaks is observed. The representative cell analysis method enabled to consider infinite lattices of different topologies without any simplifying assumptions. Fracture toughness evaluated for both in plane fracture modes helps to explain the obtained propagation paths.

FRACTURE SCENARIO

Consider an infinite plane beam lattice possessing 2D translational symmetry. The elastic response of the lattice is defined by the bending and the axial stiffnesses of the beams rigidly connected at the lattice nodes. The lattice is subjected to remote tensile loading generating a periodic stress state. Increasing the remote loading leads to rods failures. If the damage is distributed single breaks one has a problem of determining the decrease in material strength [2]. Another limiting situation is when a sufficiently large number of broken rods produce a crack on a macrolevel. In this case the fracture properties of cellular material can be investigated by the use of semi-infinite crack model [1,3,6]. The goal of the present work is to study the complete process of crack nucleation from a single break.

In order to examine the nucleation phenomenon we have to accept some failure criterion. It is assumed that when the maximum tensile stress in a beam approaches some critical value brittle fracture of the beam takes place. This value is approximately equal to the modulus of rupture σ_{fc} for the bulk material [3]. After the first rod failure the stresses around the flaw redistribute. The analysis of the stress state shows which of the neighboring rods will fail next, then the lattice with two rods missing is examined. Continuation of this procedure produces a flaw pattern. For all the considered layouts with triangular, square and heagonal cells this flaw becomes stable after a sufficiently large number of broken rods and is composed of one or two macrocracks.

ANALYSIS METHOD

In the literature there are two main approaches to the analysis of infinite beam lattice failure[3]. The first one is to consider a sufficiently large finite domain and to employ the finite element method. The second approach relates the infinite lattice to an elastic continuum using some simplifying assumptions. Herein we use another way and carry out the exact analysis of infinite lattice structure. This is done by means of the reprecentative cell method based on the discrete Fourier transform [4,5]. This method enables to reduce the analysis of arbitrary loaded structure possessing translational symmetry to the analysis of a single representative periodicity cell. For the triangular and hexagonal layouts this cell consists of three beams and for the square layout of two beams. The boundary problem for the cell is formulated with respect to the complex valued displacement transforms. Their values at the opposite cell boundaries are related by Born - Von Karman type boundary conditions.

The absence of the translational symmetry in the lattice structures with several broken beams is overcomed by introducing fictitious forces applied at the ends of these beams. The amplitudes of the forces are obtained at each step from a linear algebraic system of $3N$ equations where N is the number of broken elements.

RESULTS

The results of numerical experiments carried out in accordance with the described fracture scenario for three different layouts are presented. Note that the triangular and hexagonal lattices on a macrolevel possess isotropic elastic effective properties. The remote loading is uniaxial tension as shown in Fig. 1. For the triangular layout there is nothing surprising: a straightlinear crack perpendicular to the loading direction is produced. In the case of honeycomb lattice the result is different: after several initial breaks a crack propagating in the direction inclined at 30 degrees with the loading direction is observed (Fig. 1b) Note, that this crack propagation path differs from the one found in a similar problem [6]. The square layout provides somewhat unexpected result. cracks propagating parallel to the applied loading direction (Fig. 1c).

In order to explain the observed propagation paths the fracture toughnesses K_{Ic} and K_{IIc} have been determined. To this end a long straightlinear crack undergoing corresponding deformation modes was considered. The accuracy of the results was verified by the comparison of the energy release rates evaluated from the lattice analysis and from the problem of a crack in a homogenized elastic continuum possessing effective elastic properties.

The obtained results reveal a significant influence of the material microstructure on the crack propagation direction and show that in cellular materials, in contrast to homogeneous ones, the condition $K_{II} = 0$ (zero Mode II stress intensity factor) can not be employed for predicting the crack propagation path.

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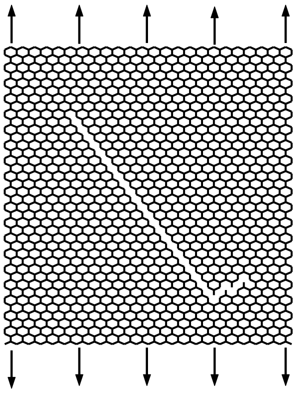
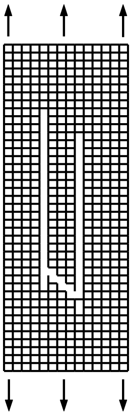


(b)



(c)

Figure 1 Cracks produced in hexagonal(b) and square cell (c) infinite lattices under uniaxial tensile remote loading.