INCOMPRESSIBLE FLOW WITH ELASTIC FILAMENTS USING MOVING OVERSET GRIDS

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Summary
We consider elastic filaments with mass that are coupled to a viscous incompressible flow using a new pressure velocity formulation. The key idea is to use thin, body-fitted grids that move and deform with moving boundaries, while using fixed Cartesian grids to cover most of the computational domain. Since the elastic boundary is always aligned with a grid line we can guarantee there is no leakage across immersed elastic boundaries: This is a major improvement over the immersed boundary and immersed interface methods.
Our approach combines the strengths of earlier moving overset grid methods for rigid body motion, and unstructured grid methods for flow-structure interactions. Large scale deformation of the flow boundaries can be handled with only locally regenerated grids that adapt to moving boundaries in a computationally efficient way. Numerical experiments are used to demonstrate the improved accuracy of the method over prior work.

INTRODUCTION

The dynamics of moving elastic interfaces coupled to a viscous flow still represents one of the most challenging numerical problems in computational fluid dynamics. Peskin's groundbreaking work in the 1970's and many developments based on the immersed boundary method since then (see Peskin & McQueen and the references therein) has lead to an improved understanding of how to solve these problems that are crucial to biological and industrial flow problems. Many problems remain, mainly due to the limited spatial resolution arising from the smearing of elastic forces to the Cartesian grid used for solving the fluid dynamic equations. The immersed interface method of LeVeque and coworkers (see, e.g., Lee & LeVeque[1] and the references therein) provides some improvement in the boundary resolution. Nevertheless, these schemes are restricted to modest Reynolds numbers (Re<300) and to flows where boundary layers do not play a significant role.
We have developed a new computational method that fixes the major shortcomings of the immersed boundary and the immersed interface method: We present a moving overset grid method that yields excellent resolution of the fine scale structure of the flow near the elastic boundary, as well as away from the boundary.

MATHEMATICAL FORMULATION AND THE NUMERICAL SCHEME

We present a new mathematical formulation of viscous flow coupled to an elastic filament. We consider viscous incompressible flow in a domain Ω that is governed by the Navier-Stokes equations

$$\mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} \tag{1}$$

$$\Delta p = \nabla u \cdot \mathbf{u}_x + \nabla v \cdot \mathbf{u}_y \tag{2}$$

where $\nabla\cdot\mathbf{u} = 0$ has been used to obtain the pressure Eq. (2). Velocity boundary conditions are specified as

$$\nabla\cdot\mathbf{u} = 0, \qquad \mathbf{u} = \frac{\partial \mathbf{x}}{\partial t} \tag{3}$$

on the filament Γ.
The elastic filament dynamics is governed by an Euler-Bernoulli beam equation[2]

$$M\frac{\partial^2\mathbf{x}}{\partial t^2} = \frac{1}{s_\alpha}\frac{\partial}{\partial\alpha}(\Lambda\mathbf{s}) - A\frac{1}{s_\alpha}\frac{\partial}{\partial\alpha}\left(\frac{\kappa_\alpha}{s_\alpha}\mathbf{n}\right) - [\![p]\!]\mathbf{n} + \nu\left[\!\!\left[\frac{\partial}{\partial n}(\mathbf{s}\cdot\mathbf{u})\right]\!\!\right]\mathbf{s} \tag{4}$$

$$\Lambda = \gamma\left(\left|\frac{\partial\mathbf{x}}{\partial\alpha}\right| - 1\right) \tag{5}$$

where the jump $[\![p]\!] = p_2 - p_1$ is across Γ, and $\mathbf{g}$ is in the direction of gravity and scaled $|\mathbf{g}| = 1$. Here α is a Lagrangian material parametrization that is equivalent to the equal arclength frame s in the reference state where the filament has length L_{ref}.
The terms in the beam Eq. (4) are, from left to right, the filament acceleration, the tension, the bending force, the pressure forcing, the viscous shear-stresses, and gravity. Note that the Lagrangian parametrization in α is different than an equal-arclength parametrization denoted here by s.
We introduce a new pressure-velocity formulation for coupling elastic interfaces with mass to a viscous incompressible flow. The pressure satisfies on the elastic filament Γ the coupling conditions

$$\frac{\partial p_1}{\partial n_1} + \frac{1}{M}(p_1 - p_2) = -\frac{1}{M}(\Lambda\kappa - A\kappa_{ss}) + \nu\,\mathbf{n}_1\cdot\nabla^2\mathbf{u}_1 \tag{6}$$

$$\frac{\partial p_2}{\partial n_2} + \frac{1}{M}(p_2 - p_1) = \frac{1}{M}(\Lambda\kappa - A\kappa_{ss}) + \nu\,\mathbf{n}_2\cdot\nabla^2\mathbf{u}_2 \tag{7}$$

These are used as boundary conditions on the pressure Poisson equation in the numerical scheme.

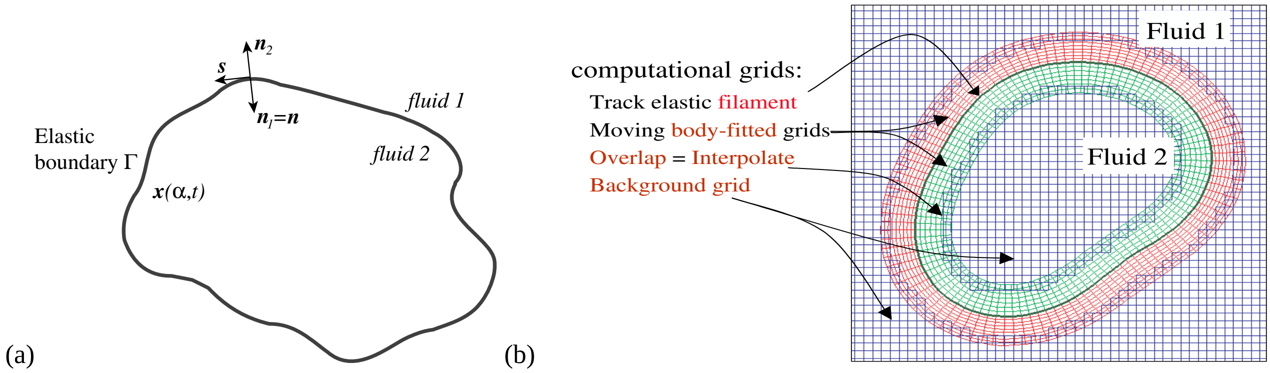

Figure 1. (a) We consider the dynamics of a closed elastic filament with mass separating two viscous incompressible fluids 1 and 2. (b) The problem is discretized with the overset grid method. Thin body-fitted grids adapt to the moving elastic filament Γ while most grids are Cartesian and stationary.

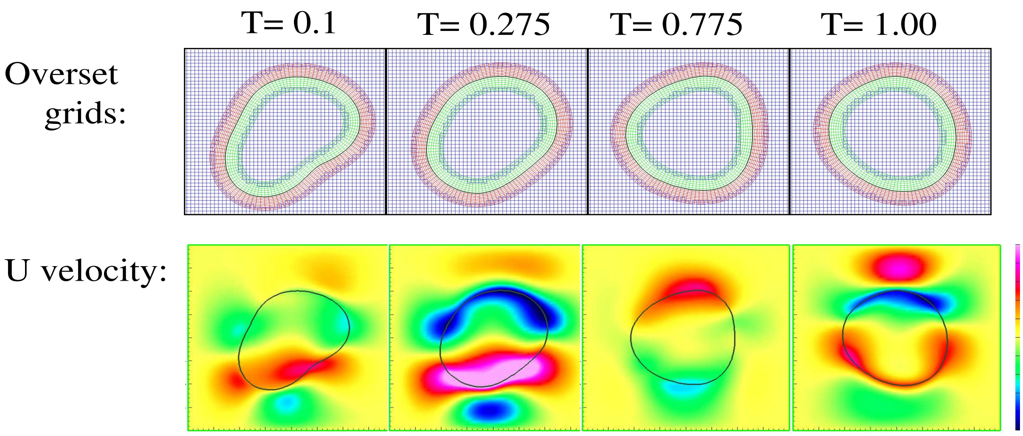

Figure 2. Simulations of a closed elastic filament relaxing under viscous and elastic forces in a closed box. The top row shows the moving overset grids at several time instances. The bottom row shows the horizontal (U) velocity component at the same time points. Notice the sloshing of the fluid that is dragging along by the elastic filament undergoing oscillations. This flow will eventually tend to a steady state with a stationary circular filament.

Very brief description of the new numerical scheme

We solve the equations of motion using a fully explicit, split-step time integrator. The spatial discretization uses curvilinear grid finite differencing on each structured component grid. The grids are coupled by interpolation conditions. The discretized pressure-Poisson equation arising from the incompressibility condition is collected into a sparse linear system which is solved by a general purpose Krylov space iterative solver such as PETSc[3]. The overset grid is regenerated each time step using the Overture framework[4], which also provides the spatial discretization.

RESULTS

We show here some preliminary results of an elastic filament that is relaxing to a circle under viscous forces and elastic forces (Fig. 2). The scheme is second order accurate in space and time, and preserves well the fluid volume enclosed by the filament. Further numerical examples will be presented at the conference.

References

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