

## PISTON IMPACT ONTO THE BOUNDARY OF TWO-LAYER FLUID

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**Summary** The plane unsteady problem of fluid flow caused by impact onto its boundary is considered. Initially fluid is at rest and occupies a lower half-plane. The fluid consists of two horizontal layers with different densities and sound velocities. The fluid flow is described within the acoustic approximation. For the special case, when impact velocity is a step function, both the velocity potential and the distribution of the pressure along the upper boundary of the fluid are obtained in the forms of series at any time instant.

### General statement of the problem

The plane unsteady problem of fluid flow caused by an impact on its boundary is considered. Initially ( $t = 0$ ) the fluid is at rest and occupies a lower half-plane ( $y < 0$ ). The fluid consists of two horizontal layers. We take the thickness of the upper layer  $\ell$  as the lengthscale and the maximal value of the boundary velocity  $V_0$  as the velocity scale of fluid flow. In dimensionless variables the line  $y = -1$  corresponds to the interface between the layers and the line  $y = 0$  to the rigid boundary of the flow region. The density of the resting fluid and its sound velocity are  $\rho_1, c_1$  in the upper layer,  $-1 < y < 0$ , and  $\rho_2, c_2$  in the lower layer,  $y < -1$ , respectively. Unsteady flow of the compressible fluid is caused by displacement of its rigid and initially flat boundary. At each time instant  $t > 0$  the boundary position is given by the equation  $y = -(V_0/c_1)f_b(x, t)$ . The function  $f_b(x, t)$  and its first derivatives are assumed to be bounded with  $f_b(x, 0) = 0$ . The ratio  $\ell/c_1$  is taken as the timescale and the product  $\rho_1 c_1 V_0$  as the scale of the hydrodynamic pressure.

We shall determine the hydrodynamic pressure along the moving rigid boundary under the following assumptions: (i) the fluid is ideal and compressible; (ii) the Mach number  $V_0/c_1 \ll 1$ ; (iii)  $c_1/c_2 \leq 1$ ; (iv) external mass forces are absent.

The fluid flow is described within the framework of the acoustic approximation provided that  $V_0/c_1 \ll 1$  and  $V_0/c_2 \ll 1$ . This approximation is valid for impact speeds well below the medium's acoustic velocity and for times in which the displacements of the rigid boundary are small compared to the thickness of the upper layer.

During the initial stage considered here displacements of the fluid particles are small compared to the lengthscale, which allows us, as a first approximation, to impose boundary conditions on the initial positions of the rigid surface ( $y = 0$ ) and the interface ( $y = -1$ ), and to linearize them and the equations of motion near the initial rest state (see [1] for details).

The linearization leads to the well-known acoustic approximation, where the liquid motion is irrotational and is described by the velocity potentials  $\phi_1(x, y, t)$  in the upper layer ( $-1 < y < 0$ ) and  $\phi_2(x, y, t)$  in the lower layer ( $y < -1$ ). The potentials  $\phi_j(x, y, t)$ ,  $j = 1, 2$  are identically zero and when  $t > 0$  satisfy the wave equations

$$\frac{\partial^2 \phi_1}{\partial t^2} = \frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial y^2} \quad (-1 < y < 0), \quad \frac{\partial^2 \phi_2}{\partial t^2} = m^2 \left( \frac{\partial^2 \phi_2}{\partial x^2} + \frac{\partial^2 \phi_2}{\partial y^2} \right) \quad (y < -1),$$

where  $m = c_2/c_1$ ,  $m \geq 1$ , the linearized boundary condition on the rigid surface

$$\frac{\partial \phi_1}{\partial y} = f(x, t) \quad (y = 0),$$

where  $f(x, t) = -(\partial f_b / \partial t)(x, t)$ , the linearized dynamic and kinematic boundary conditions on the interface ( $k = \rho_1/\rho_2$ )

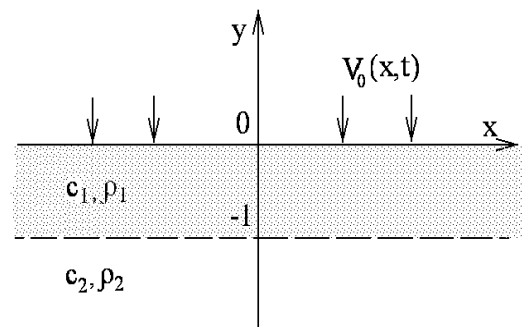
$$\phi_2 = k\phi_1, \quad \frac{\partial \phi_2}{\partial y} = \frac{\partial \phi_1}{\partial y} \quad (y = -1).$$

The distribution of hydrodynamic pressure along the rigid boundary,  $y = 0$ , is given in the non-dimensional variables as  $p_1(x, 0, t) = -(\partial \phi_1 / \partial t)(x, 0, t)$  within the acoustic approximation. Below we determine the potential  $\phi_1(x, 0, t)$  along the rigid boundary and differentiate it with respect to time thereafter to derive the final formula for the hydrodynamic pressure distribution.

### Method of solution

We use the Fourier transform with respect to the horizontal coordinate  $x$  and the Laplace transform with respect to time  $t$ . It allows us to present the solution of the problem in the form

$$\phi_1(x, 0, t) = \phi_1^{(0)}(x, 0, t) + 2 \sum_{n=1}^{\infty} q_{\infty}^n \phi_1^{(n)}(x, 0, t) + 2 \sum_{n=1}^{\infty} \phi_{1n}(x, 0, t).$$



Each element in these series are obtain separately. We consider below important particular case, when impact velocity is a step function. Then  $f_x(x)H(t) = \sum_{N=1}^{P_*} S_N \delta(x - x_N)H(t)$ ,  $S_N = f(x_N + 0) - f(x_N - 0)$ , where  $H(t) = 1$  if  $t \geq 0$  and  $H(t) = 0$  if  $t < 0$ . It was obtain (see [1] for details)

$$\frac{\partial \phi_1^{(0)}}{\partial t}(x, 0, t) = f_0(x)H(t) - \frac{1}{2} \sum_{N=1}^{N_*} S_N \frac{x - x_N}{|x - x_N|} H(t - |x - x_N|) + \frac{1}{\pi} \sum_{N=1}^{N_*} S_N \arcsin\left(\frac{x - x_N}{t}\right) H(t - |x - x_N|).$$

$$\frac{\partial \phi_1^{(n)}}{\partial t}(x, 0, t) = \frac{\partial \phi_2^{(0)}}{\partial t}(x, 0, \sqrt{t^2 - 4n^2}) H(t^2 - 4n^2) \quad (n \geq 1),$$

$$\frac{\partial \phi_{1n}^{(2)}}{\partial t}(x, t) = \frac{1}{\pi} \sum_{N=1}^{N_*} S_N H(x_N - x - \sqrt{t^2 - 4n^2}) H(x - x_N + a_n(t, m)) \int_{\lambda_1(x_N - x, t)}^m \frac{\sin[2n\varphi(\lambda)]}{\sqrt{\lambda^2 - 1}} d\lambda H\left(t - \frac{2nm}{\sqrt{m^2 - 1}}\right) -$$

$$-\frac{1}{\pi} \sum_{N=1}^{N_*} S_N H(x - x_N - \sqrt{t^2 - 4n^2}) H(x_N - x + a_n(t, m)) \int_{\lambda_1(x - x_N, t)}^m \frac{\sin[2n\varphi(\lambda)]}{\sqrt{\lambda^2 - 1}} d\lambda H\left(t - \frac{2nm}{\sqrt{m^2 - 1}}\right),$$

where  $\lambda(z) = \sqrt{1 + (m^2 - 1)z^2}$ .

### Numerical results

Calculations were performed for the case, with  $f(x) = 0$ ,  $x \in (-\infty, -4) \cup (4, \infty)$  and  $f(x) = 1$ ,  $x \in [-4, 4]$ . This corresponds to the problem of piston entering two-layer liquid through its unconformable boundary. We consider following cases: (1) impact on compressible liquid of finite depth,  $k \ll 1$  (in calculation  $k = 0.001$ ,  $m = 1$ ) (curve 1); (2) impact on compressible liquid of infinite depth,  $k = 1$ ,  $m = 1$  (curve 2); (3) incompressible fluid in the lower layer,  $m \gg 1$  ( $m = 300$ ,  $k = 1$ ) (curve 3); (4) both fluid are compressible but different  $m = 3$ ,  $k = 0.9$  (curve 4); (5) both fluid are compressible  $m = 10$ ,  $k = 0.7$  (curve 5).

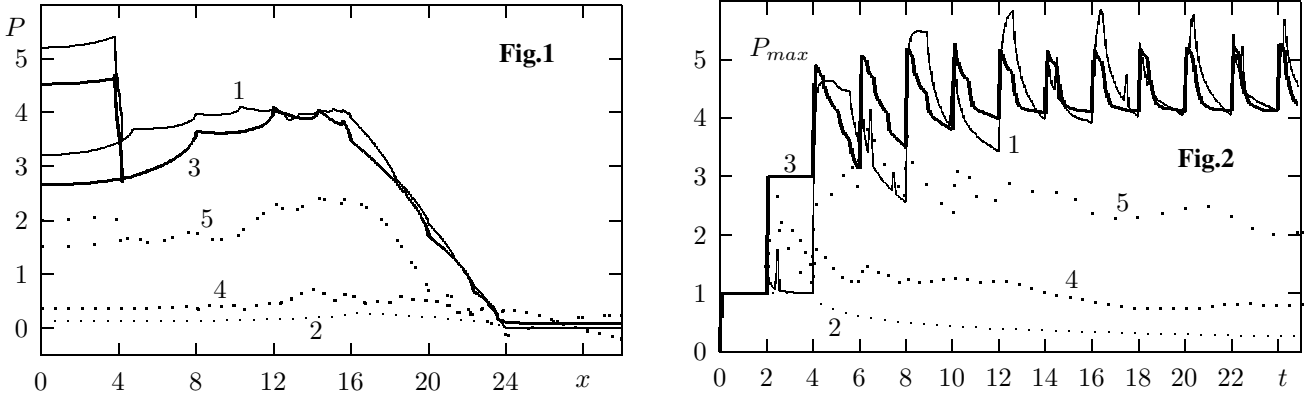


Figure 1 presents distributions of the pressure along the boundary at the moment  $t = 20.01$  for  $x > 0$ . One can see that the pressure expands along the boundary and reaches maximum value outside of impact region for cases (2),(4),(5). For cases (1) and (3) and times  $t = 2n + \Delta$  (times, when reflected waves come to the upper boundary) maximums occur inside the impact region. Figure 2 depicts the pressure maximums depending on time. One can see that the maximum of the pressure decreases for cases (2),(4),(5). For case (2) the pressure peak moves along the boundary at sound speed. For cases (4),(5) this velocity a little bit lower. Maxima of the pressure keep the values long time, if  $c_1$  and  $c_2$  or  $\rho_1$  and  $\rho_2$  very different (cases (1) and (3)).

### Conclusions

Model of the impact onto the boundary of two-layer fluid in acoustic approximation is presented. For the case, when impact velocity is a step function, both the velocity potential and the distribution of hydrodynamic pressure along the upper boundary of the fluid are obtained in the forms of series at any time instant. With the help of this model the problem of the piston impact is analyzed. The problem is to illustrate the scopes of the presented method. The analysis can be extended to the such important case as impact of fluid with the bubbly fore-part.

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### References

- [1] Korobkin A.A.: Impact onto the boundary of two-layer fluid. 2003, *under preparation*.