ON A FLUID-ELASTIC ISOTROPIC CUSPED PLATE INTERACTION PROBLEM

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Summary The purpose of this paper is to determine transmission conditions for thin elastic cusped plate-incompressible viscous fluid interaction problems and to investigate the problem of vibration of a plate caused by the flow of the fluid.

For the last decades the direct and inverse problems connected with the interaction between difference vector fields have received much attention in the mathematical and engineering scientific literature and have been intensively investigated. They arise in many physical and mechanical models describing the interaction of two different media where the whole process is characterised by a vector-function of dimension k in one medium and by a vector-function of dimension n in another one (for example, fluid-structure interaction where a streamlined body is an elastic obstacle, scattering of acoustic and electromagnetic waves by an elastic obstacle, interaction between an elastic body and seismic waves, etc.).

A lot of authors have considered and studied in detail the direct problems of interaction between an elastic isotropic body occupying a bounded region $\overline{\Omega}$ with a three-dimensional elastic vector field to be defined, and some isotropic medium (say fluid) occupying the unbounded exterior region, the compliment of Ω with respect to the whole space, where a scalar field is to be defined. Our aim is to consider interaction problems when the profile of an elastic part is cusped one on some part or on the whole boundary (for cusped plates see [1],[2],[3]).

We consider the interface problem of the interaction of a flow of the incompressible viscous fluid and of a plate whose variable flexural rigidity is given by the following expression

$$D(x_2) = D_0 x_2^{\alpha}(l - x_2)^{\beta}, \;\; D_0, l = \text{const} > 0, \;\; \alpha, \beta = \text{const} \geq 0,$$

caused by the thickness

$$2h(x_2) = h_0 x_2^{\alpha/3}(l - x_2)^{\beta/3}, \;\; h_0 = \text{const} > 0.$$

In case $\alpha^2 + \beta^2 > 0$ plates are called cusped plates.

Let the flow of the fluid be independent of x_1, parallel to the plane $0x_2x_3$, i.e. $v_1 \equiv 0$, and generating bending of the plate. We suppose that at infinity we have the following conditions

$$p(x_2, x_3, t) \to p_\infty(t), \;\; \text{when} \;\; |x| \to \infty,$$

$$v_2(x_2, x_3, t) = O(1), \;\; v_3(x_2, x_3, t) \to v_{3\infty}(t), \;\; \text{when} \;\; |x| \to \infty,$$

where $v := (v_2, v_3)$ is a velocity vector of the fluid, $p(x_2, x_3, t)$ is a pressure, and $v_{3\infty}(t)$, $p_\infty(t)$ are given functions.

Let introduce the following notations

$$I := \{[0, l] \times 0\},$$
$$\Omega^f := \left\{x_1, x_2, x_3 : x_1 = 0, \; x := (x_2, x_3) \in R^2 \backslash I\right\}.$$

If the middle plane of the plate lies in the plane $0x_1x_2$ and the flow of moving fluid involves bending of the plate then transmission conditions could have the form:

$$\sigma^f_{N3}\left(x_2, \overset{(+)}{h}(x_2), t\right) - \sigma^f_{N3}\left(x_1, x_2, \overset{(-)}{h}(x_2), t\right) = q(x_2, t),$$

$$v_3\left(x_1 - \overset{(+)}{h}(x_1, x_2)w_{,1}(x_1, x_2, t), x_2 - \overset{(+)}{h}(x_1, x_2)w_{,2}(x_1, x_2, t), \overset{(+)}{h}(x_1, x_2) + w(x_1, x_2, t), t\right)$$

$$= v_3\left(x_1 - \overset{(-)}{h}(x_1, x_2)w_{,1}(x_1, x_2, t), x_2 - \overset{(-)}{h}(x_1, x_2)w_{,2}(x_1, x_2, t), \overset{(-)}{h}(x_1, x_2) + w(x_1, x_2, t), t\right) = \frac{\partial w(x_1, x_2, t)}{\partial t},$$

$$x_2 \in]0, l[, \;\; t \geq 0,$$

(the first of the last pair of equalities is valid since deflection of plate w is independent of x_3), where σ_{ij} is a stress tensor of a fluid, $q(x_1, x_2, t)$ is a lateral load, $w(x_1, x_2, t)$ is a deflection of the plate, $\overset{(+)}{h}(x_2)$ and $\overset{(-)}{h}(x_2)$ are upper and lower surfaces of the plate, in general $2h(x_2) := \overset{(+)}{h}(x_2) - \overset{(-)}{h}(x_2)$.

Further, in case of an incompressible fluid, since the flow of the fluid generates bending of the plate, a transmission condition for the pressure can be written as follows

$$\begin{aligned} &- p(x_2, \overset{(-)}{h}(x_2), t)\cos(\overrightarrow{n}(x_2, \overset{(-)}{h}(x_2)), x_3) - p(x_2, \overset{(+)}{h}(x_2), t)\cos(\overrightarrow{n}(x_2, \overset{(+)}{h}(x_2)), x_3) \\ &- 2\mu\left(\frac{\partial v_2(x_2, \overset{(-)}{h}(x_2), t)}{\partial x_2}\cos(\overrightarrow{n}(x_2, \overset{(-)}{h}(x_2)), x_3) + \frac{\partial v_2(x_2, \overset{(+)}{h}(x_2), t)}{\partial x_2}\cos(\overrightarrow{n}(x_2, \overset{(+)}{h}(x_2)), x_3)\right) \\ &= q(x_2, t), \quad x_2 \in]0, l[, \quad t \geq 0. \end{aligned}$$

We consider the case when the motion of the fluid is sufficiently slow, i.e., v_j and $v_{j,k}$ $(j, k = 2, 3)$ are so small that linearized of Navier-Stokes equations can be applied.

In the solid part the have the following equation

$$(D(x_2)w,_{22}(x_2, t)),_{22} = q(x_2, t) - 2\rho h(x_2)\frac{\partial^2 w(x_2, t)}{\partial t^2}, \qquad 0 < x_2 < l, \tag{1}$$

where $w(x_2, t)$ is a deflection of the plate, ρ is a density of the plate, by $w,_i$ we denote $w,_i := \partial w/\partial x_i$. The setting of boundary conditions at the plates end depends on the geometry of sharpenings (i.e., on α, β) of plates ends, while the setting of initial conditions is independent of them (see [3]-[5]).

Remark 1 *If the plate thickness is sufficient small, we can assume that:*
1. the fluid occupies $R^2 \backslash I$*;*
2. the plate occupies I *(its geometry depending on the thickness is taken into account in the coefficient of the bending equation);*
3. Transmission conditions for $v_3(x_2, x_3, t)$ *and* $v_2(x_2, x_3, t)$ *can be written in the following form (see [4]-[6])*

$$v_2(x_2, 0, t) = 0, \quad v_3(x_2, 0, t) = \frac{\partial w(x_2, t)}{\partial t}, \quad x_2 \in]0, l[, \quad t \geq 0.$$

References

[1] **Vekua, I.N.**: Shell Theory: General Methods of Construction. *Pitman Advanced Publishing Program, Boston-London-Melbourne* 1985.

[2] **Jaiani, G.V.**: Elastic bodies with non-smooth boundaries–cusped plates and shells. *ZAMM-Zeitschrift fuer Angewandte Mathematik und mechanik* **76**: 117-120, 1996, Suppl. 2.

[3] **Chinchaladze, N.**: Vibration of a plate with two cusped edges. *Proceedings of I. Vekua Institute of Applied Mathematics of Tbilisi State University* **52**: 30-48, 2002.

[4] **Chinchaladze, N.**: Bending of an isotropic cusped elastic plates under uction of an incompressible fluid. *Reports of the Seminar of I. Vekua Institute of Applied Mathematics* **28**: 52-60, 2002.

[5] **Chinchaladze, N., Jaiani, G.**: On a cusped elastic solid-fluid interaction problem. *Applied Mathematics and Informatics* **6, 2**: 25-64, 2001.

[6] **Chinchaladze, N.**: A cusped elastic plate-ideal incompressible fluid interaction problem. *Reports of the Seminar of I. Vekua Institute of Applied Mathematics* **28**: 31-39, 2002.