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Affine Symmetry in Mechanics of Discrete and Continuous Systems

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Discussed are the theory and applications of affine invariance in dynamics of multiparticle systems and continuous media. Traditional invariance and objectivity principles are based on Euclidean geometry of the physical space and its isometry groups (translations and rotations). Among all constrained mechanical systems rigid bodies play a distinguished role both in macroscopic mechanics and in theory of microstructured bodies like, e.g., Cosserat continua. Similarly, in fundamental physics elementary particles and fields are classified in terms of representations of isometry groups (Euclidean and Lorentz groups). Models of collective and internal degrees of freedom based on affine geometry (rigid rotations and homogeneous deformations)are also used in various mechanical and physical problems, e.g., in macroscopic elasticity, mechanics of micromorphic continua (Eringen), molecular and nuclear dynamics and astrophysics. However, it is only kinematics, but not dynamics that is affinely invariant. Unlike this, we develop mathematical models of affine degrees of freedom the dynamics of which is ruled by affine group. In a sense, this may be considered as a discretization of the Arnold, Marsden and others model of the ideal fluid as a Hamiltonian system on the diffeomorphism group. In particular, we discuss geodetic models, where the dynamics of elastic vibrations is encoded in an appropriately chosen kinetic energy form invariant under the affine group. Independently of the mathematical beauty of such models and in spite of their apparently academic character, they seem to be applicable in certain problems of microstructured media, the dynamics of defects in solids, nanomechanics and in fundamental physics like, e.g., the droplet model of nuclei. Certain aspects of our results are relevant for the theory of completely integrable mechanical systems.

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