ON THE VOLUMETRIC GROWTH OF BINARY SOLID-FLUID MIXTURES

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Summary
This work aims at investigating the possibility of modelling the volumetric growth of a binary solid-fluid mixture by describing the time evolution of the solid stress-free configuration, within the context of biomechanical perspectives in rational mixture theories.

A GROWING SOLID INFUSED WITH A FLUID

The volumetric growth of living tissues, here regarded as binary solid-fluid mixtures, generally occurs through cell division (or death), cell enlargement (or shrinkage), and secretion (or resorption) of extracellular matrix (Taber [17]).

A binary mixture can be macroscopically described as a couple of body manifolds, embedded into the three-dimensional Euclidean space, so as to share a smooth region of the physical environment while undertaking independent motions (see e. g. Atkin and Craine [1], Bowen [2], Nunziato and Walsh [12], Rajagopal and Tao [15], Truesdell [18]).

Moreover, a mathematical theory of solid-fluid mixtures provides a suitable framework for the development of a consistent macroscopic theory of porous solids, partially or totally saturated with compressible or incompressible fluids (Bowen [3, 4], Quiligotti et al. [13], Wilmanski [19, 20]), which may be employed to model relevant aspects of bone mechanics (cf. Cowin [5, 6]).

In order to describe the volumetric growth of both incompressible and compressible elastic biological materials (Taber [17], Klisch and Van Dyke [10]) and the remodeling of living tissues (see e. g. Humphrey and Rajagopal [9]), thermodynamically admissible constitutive prescriptions for additional mass supply densities need to be proposed, whenever mass production and mass resorption are considered.

Extending the pioneering proposal put forward by Rodriguez, Hoger and McCulloch [16] to binary solid-fluid mixtures, the *bulk growth* of a living soft tissue is here regarded as the *time evolution of its stress-free configuration* (cf. Di Carlo and Quiligotti [14], Epstein and Maugin [7]), described by a smooth (but geometrically noncompatible) tensor field on the reference configuration. The geometrical incompatibility of this additional descriptor (cf. Lee [11]) is mostly due to the fact that if we allowed any given body element to grow independently of neighbouring particles in the absence of external applied load, we would generally find out that body elements may no longer be geometrically compatible, after growing. Hence, to make them fit together again, it may be necessary to deform them, giving rise to a residual stress field whose existence in biological tissues has been experimentally observed and investigated (see, for instance, Fujie, Yamamoto et al. [8], Yasuda and Hayashi [21]).

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