

Nonlinear Oscillators with Time Delays

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Because of its importance for various fields (technology, physics, electronics, biology, medicine) and also for controlling the chaos, the theory of delay differential equations has been violently developed during the last 25 years. Here we present the theoretical as well as numerical results for a nonlinear damped oscillator, with a small part of the restoring force retarded in time. It appears that the dynamics of such a system is quite complex. Depending on the retardation time t a number of Hopf bifurcations as well as a number of various chaotic regimes appears. Also different routes to chaos are observed. For several cases we reconstructed strange attractors and estimated their correlation dimensions as well as Lapunow exponents. For a small damping, the time of the first Hopf bifurcation is proportional to the damping in the system. This demonstrates that hamiltonian systems can be unstable with respect to perturbations containing terms with time delay. The equation considered here was proposed as a simplest possible model of keyhole instabilities observed during the laser welding. That is to say, the thin channel formed by metallic vapours in the molten material is performing irregular chaotic oscillations.

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