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PARAMETRIC INSTABILITY AND CHAOS IN THE SIMPLE PORTAL UNDER GROUND MOTIONS

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Summary The response of an elastic frame, with inertial nonlinearities (single degree of freedom) due to vertical and horizontal ground motion, are investigated. The interactions between horizontal and vertical ground motions are considered. The instability regions are investigated numerically. The results of the numerical analysis reveal periodic, quasi–periodic and chaotic motions, as shown in Poincare's maps.

INTRODUCTION

The calculation of the construction behavior during the earthquakes usually encompasses the influence of only one horizontal component. It is known that periodical vertical motions of the ground have a parametric influence on the horizontal motions. In this paper ground motion is idealized as harmonic. The horizontal motions are mathematically determined by the Mathieu's homogeneous differential equation. All the solutions of this equation are determined in a two–dimensional space with areas of instability. The influence of vertical motions is extremely significant if their frequency is within the main area of instability (parametric resonance). The paper explores numerically the areas of unstable oscillations of the models of simple, ideally elastic frame caused by simultaneous interaction of the horizontal and vertical motions of different frequencies. For real problems with many degrees of freedom, similar analysis is practically impossible.

EQUATIONS OF MOTIONS

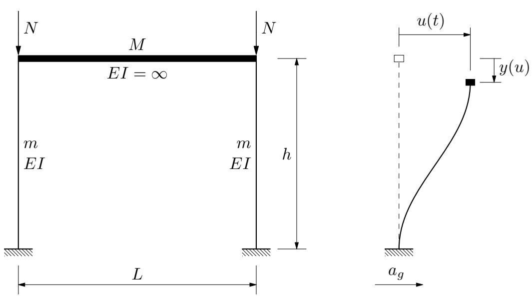


Figure 1: Simplified portal frame

The motion equations of the simple, elastic frame are deduced for simultaneous horizontal and vertical earthquake activity. A non–linear inertia $M\ddot{y}$ in linear form is included. It has an additional effect on selfcreating parametric oscillations. All variables have been re–scaled into non–dimensional form: the time by use natural frequency Ω, and displacements, by use of the hight of the column h. The usual non–dimensional coefficient: $\mu = N_t/[2(N - N_e)]$ and ratio of the frequencies of the vertical and horizontal motions to the base frequency Ω, 2β and ξ, are introduced. The coefficients c and A are introduced also: $c = 252/(175 + 130m/M)$, $A = a_g/(\Omega^2 h)$. Using these relations the equations of motions take the form (1)

$$\begin{aligned} \dot{w} + (1 - 2\mu\cos 2\beta t)u + c(\dot{u}^2 + u\,\dot{w})u &= -A\cos\xi t. \\ \dot{u} - w &= 0. \end{aligned} \qquad (1)$$

The values of the constants c and A are bounded by physical meaning on the value area, as shown in Table 1. and 2.

M/m	c
0	1.44
0.5	1.05
0.9	0.86

Table 1.

Ω	a_g	h	A
31.41 (for $N = 0$	$0.1g$	4.00	0.0002 (min. value)
⋮	⋮	⋮	⋮
0.468 (for $N = 0.8N_e$)	$0.5g$	2.00	5.0 (max. value)

Table 2.

REGIONS OF INSTABILITY

The areas of stability of solutions of the equation (1) are investigated in a Liapunov sense, e.g. by numerical integration with checking of the energy increase over time. Depending on the values of the μ, c and A parameters, equations (1) describe either linear or non–linear forced oscillations (horizontal ground motions only), linear and non–linear homogeneous parametric oscillations (vertical ground motions only) and linear or non–linear parametric oscillations in the interaction with forced oscillations.

Non–linear forced oscillations

The areas of stable oscillations depend on the value of the amplitude A, the ratio of the frequencies ξ, and the coefficient c ($\mu = 0$). The numerical integration shows a region of the main unstable resonance area (linear) and the secondary (non–linear) area with higher values of amplitude A. The coefficient c affects the non–linear intensity, and therefore broadens the instability area and reduces the value A, where the secondary area appears. For the future, we assume $c \approx 1$ (from Table 1.). Non–linear influence for the value $\mu = 0$ is shown on Figure 2.

Parametric oscillations

The area of nonlinear parametric resonance instability indicates nonlinear effects at lower coefficient value μ, as anticipated (Figure 3).

Interaction of the horizontal and vertical ground motions

Simultaneous action of the horizontal and vertical motions depends on the value of the amplitude A. The influence on unstable regions is shown for $\beta = \xi$ (Figure 4, 5 and 8). The influence of the vertical and horizontal motion frequencies ratio on the areas of instability, for the low values of the amplitude A, is shown on the Figure 11. Poincare's maps show the responses within each of the instability area. Depending on the parameter value, the response is either periodic, quasi–periodic or chaotic. Just a few interesting solutions are shown on Figure 6, 7, 9, 10, 12 and 13.

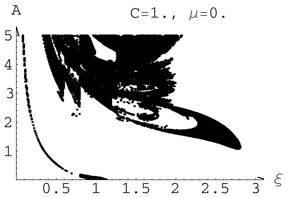


Figure 2.

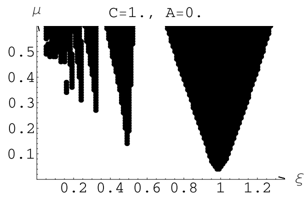


Figure 3.

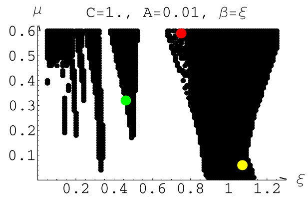
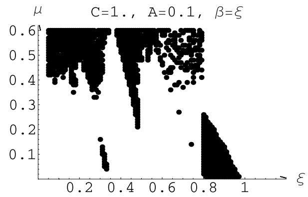


Figure 4.



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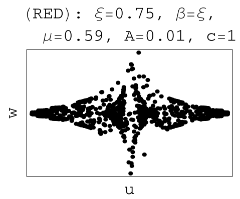
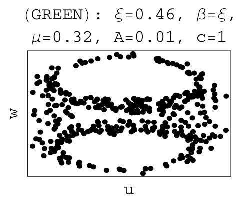


Figure 6.



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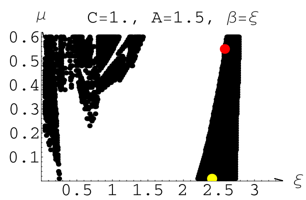

Figure 8.

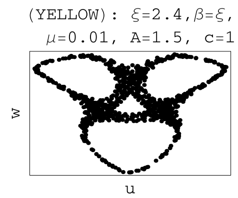

Figure 9.

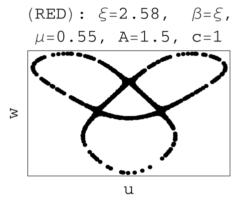

Figure 10.

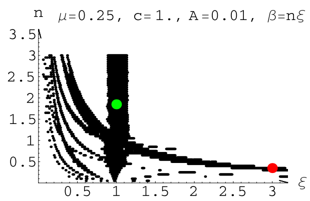

Figure 11.

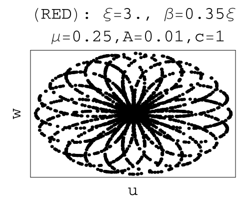

Figure 12.

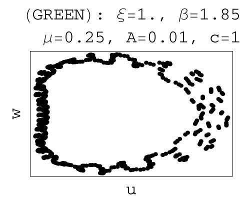

Figure 13.

CONCLUSIONS

It is to be concluded that the standard seismic calculations have been carried out with too much simplifications. On the other hand, the real structural analysis with many degrees of freedom is too complex. Even the model with a single degree of freedom indicates a chaotic response. Further investigations that would enable sufficiently safe but simple seismic calculations should be undertaken.

References

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