Fourier spectrum representation of vector multipole field and its application in wave scattering in elastic half-space

Chau-Shioung Yeh*, Tsung-Jen Teng**, Po-Jen Shih***

*Professor, Department of Civil Engineering, and Institute of Applied Mechanics, National Taiwan University,Taipei, Taiwan

** Research Fellow, Nation Center for Research on Earthquake Engineering, Taipei, Taiwan

*** Graduated Student, Department of Civil Engineering, National Taiwan University, Taipei, Taiwan

Summary This paper presents the stress concentration of a cavity which embedded in elastic half space and impinged by Goodier-Bishop incident wave. When the frequency approaching zero, Goodier-Bishop wave reduces to a field under a uniform tension along the direction parallel to the flat surface. We derive the reflective wave from the free flat surface through the Fourier spectrum representation include in the development of the transition matrix. Then the transition matrix approach is applied to determine the scattered field.

GOODIER-BISHOP INCIDENT WAVE

Many investigators have studied the dynamic stress concentration of a cavity in an elastic infinite space. The historical development and the related reference can be found in the monograph of Pao and Mow (1973). However, very little progress has been achieved in the investigation of dynamic concentration in three-dimensional elastic half-space. In this paper, we present the solution for the dynamic stress concentration of a spherical cavity in elastic half-space subjected to two sets of standing Goodier-Bishop wave with circular frequency ω as shown in Fig. 1; the one travels in *x-z* plane and is specified by the dilatational potential

$$\begin{aligned} \Phi_x^{i+r} &= 2A_0[ik_p(z+h)+4\sqrt{\kappa^2-1}/(\kappa^2-2)^2]\cos k_p x, \\ \Psi_x^r &= 4i\,A_0/(\kappa^2-2)\ e^{-ik_p\sqrt{\kappa^2-1}(z+h)}\sin k_p x, \end{aligned} \tag{1}$$

and the other travels in *y-z* plane and is specified (see Fig. 2)

$$\begin{aligned} \Phi_y^{i+r} &= 2A_0^*[ik_p(z+h)+4\sqrt{\kappa^2-1}/(\kappa^2-2)^2]\cos k_p y, \\ \Psi_y^r &= 4i\,A_0^*/(\kappa^2-2)\ e^{-ik_p\sqrt{\kappa^2-1}(z+h)}\sin k_p y, \end{aligned} \tag{2}$$

where $A_0^* = (2-\kappa^2)/[2(\kappa^2-1)]\,A_0$, $A_0 = -[(\kappa^2-1)^{1/2}(\kappa^2-2)^2\,T\,] \big/ [4\,\mu\,\kappa^2\,k_p^2(3\kappa^2-4)\,]$, $\kappa = c_p/c_s$, and the wave speed $C_p = \sqrt{(\lambda+2\mu)/\rho}$, $C_s = \sqrt{\mu/\rho}$ in which ρ denotes mass density, λ and μ are Lamé constants. Indeed, the Goodier-Bishop wave is a plane dilatational wave propagating in grazing incidence and satisfies the traction-free conditions at the flat surface. It generates a reflective *Pz*-wave as well as a reflective *SV*-wave in the direction of the critical angle. Note that the summation of Eqs. (1) and (2) yields that as $k_p \to 0$ the free field normal stress τ_{xx}^f approaches a uniform tension T and the other components vanish.

TRANSITION MATRIX IN ELASTIC HALF SPACE AND FOURIER SPECTRUM REPRESENTATION

On the basis of the transition matrix method approach, we can relate the scattering coefficients $\mathbf{c}$ to the coefficients of incident wave $\mathbf{a}$, i.e.,

$$\mathbf{c} = [\mathbf{I}-\mathbf{TP}]^{-1}\mathbf{T}\,\mathbf{a} \equiv \mathbf{T}^*\,\mathbf{a}, \tag{3}$$

where $\mathbf{I}$ denotes a unit matrix and $\mathbf{T} = -\hat{\mathbf{Q}}\,\mathbf{Q}^{-1}$ is the conventional transition matrix for the elastic infinite space (Pao, 1978). The notation $\mathbf{T}^* = [\mathbf{I}-\mathbf{TP}]^{-1}\mathbf{T}$ is the one for elastic half-space. It should be mentioned that $\mathbf{P}$ denotes the rescattering matrix which implies the rescattering phenomena of reflective wave from the flat surface. The transition matrices $\mathbf{T}$ and $\mathbf{P}$ are developed by applying Betti's third identity to the basis functions and the true field. To put it briefly, $\mathbf{P}$ matrix is formulated by applying the reciprocal theorem combined with the basis function and the rescattering wave (reflected from the free flat surface)

$$\mathbf{P} = \iint_S [\,\mathbf{t}\cdot\mathbf{u}^{rs} - \mathbf{t}^{rs}\cdot\mathbf{u}\,]\,dS, \tag{4}$$

in which the set ($\mathbf{t}, \mathbf{u}$) denotes the spherical vector basis functions Pao (1978) and the rescattering set ($\mathbf{u}^{rs}, \mathbf{t}^{rs}$) denotes the reflective wave from the free flat surface. The rescattering term, in general, could be presented as Fourier spectrum representation (on Cartesian coordinate) and be written as the following form,

$$\mathbf{u}^{rs} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{\mathbf{u}}^{rs} e^{-\sqrt{k_x^2+k_y^2-k^{*2}}\,(z+2h)-ik_x x-ik_y y}\ dk_x dk_y, \tag{5}$$

in which wave number k^* may be k_p or k_s and h denotes the depth of the cavity. In above equation, the double integrals of Fourier spectrum can be calculated by applying the modified steepest decent path method (Yeh, *et al.* 2000).

CONCLUSIONS

In this paper, we adopt the Goodier-Bishop wave to simulate a standing incident wave for modelling the uniform tension surrounding. The stress concentration of a spherical cavity in elastic half-space subjected to this standing wave calculated by employing the transition matrix for half-space and shown in Fig. 3. These results present the features of the influence of the interaction between the ground surface and a shallow embedded cavity.

References

[1] Goodier, J. N. and R. E. D. Bishop (1952), "A note on critical reflections of elastic waves at free surface", *Journal of Applied Physics*, 23(1), pp. 124-126.

[2] Mindlin, R.D. (1960), "Waves and vibrations in isotropic elastic plates," in J.N. Goodier and N.J. Hoff (eds.), *Structural Mechanics*, Pergamon Press, Oxford, pp. 199-232.

[3] Pao, Y. H. and C. C. Mow (1973), *Diffraction of elastic waves and dynamic stress concentrations*, Crane, Russak & company Inc. US.

[4] Pao, Y. H. (1978) " Betti's identity and transition matrix for elastic waves," *J. Acoust. Soc. Am.*, 64(1), pp. 302-310.

[5] Yeh, C. S., T. J. Teng, and W. I. Liao(2000), "On Evaluation of Lamb's Integrals for Waves in a Two-dimension Elastic Half-space," *The Chinese Journal of Mechanics*, Vol. 16, No. 2, 109-124

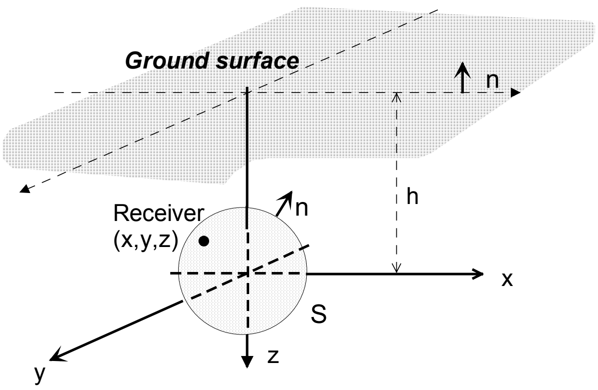
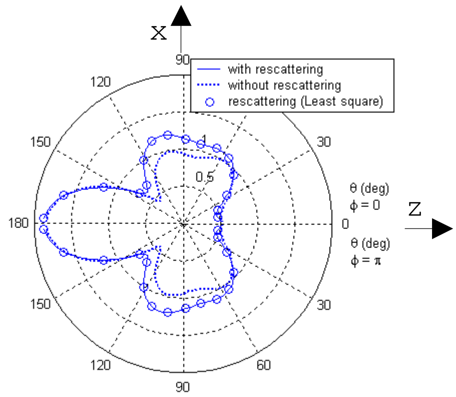


Figure 1. The coordinate system for the geometric inclusion.



(a)

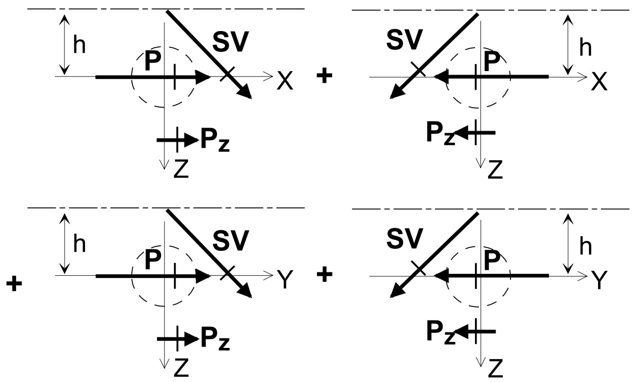


Figure 2. A diagram for the standing Goodier-Bishop stress wave.

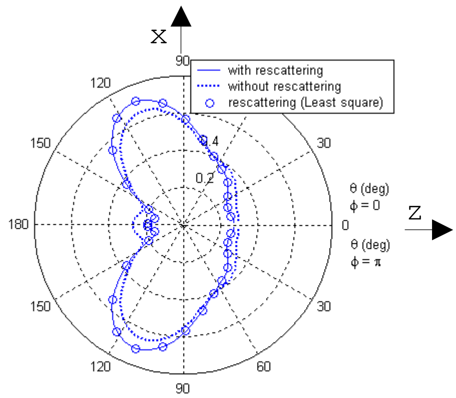


Figure 3. The influence of the rescattering through describing the hoop stress around the meridian on x-z plane ($\theta = 0 \sim \pi$ and $\phi = 0,\ \pi$) of the shallow buried cavity $k_s h = 2$ at frequency $k_s a = 1$ and (a) for $|\tau_{\theta\theta}|/T$ (b) for $|\tau_{\phi\phi}|/T$ (solid line for rescattering case, dotted line for without rescattering case and circle symbol for results of the least square method testing)