

## LIFT FORCE ON BUBBLES AND PARTICLES IN A ROTATING CYLINDER

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**Summary** We report on lift and drag coefficient measurements of bubbles and particles in a vortex flow. The Strouhal number  $Sr$  and Reynolds number  $Re$  are  $0.1 < Sr < 1$  and  $0.01 < Re < 100$  based on the typical bubble radius  $R_b \approx 1$  mm. An increased drag is found in accordance with numerical experiments in linear shear flow. Negative lift coefficients are found for  $0.1 < Re < 3$ .

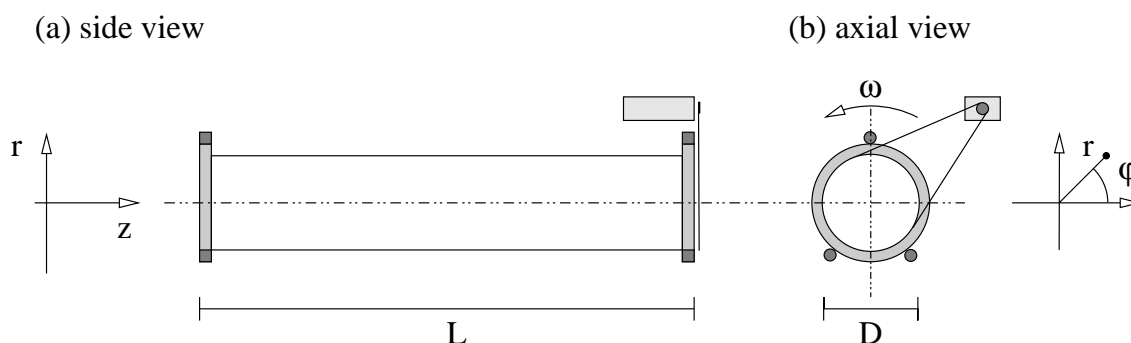
## EXPERIMENT

The experimental setup is shown in Fig. 1. A cylinder of 500 mm length and 100 mm inner diameter is filled with a glycerin–water mixture. It is rotated horizontally with an angular velocity  $\omega$  in the range of  $0 \dots 40$  rad s<sup>-1</sup>. The Strouhal number  $Sr$  and Reynolds number  $Re$  are defined by

$$Re = \frac{2R_b|\mathbf{v} - \mathbf{U}|}{\nu}, \quad \text{and} \quad Sr = \frac{2R_b\omega}{|\mathbf{v} - \mathbf{U}|},$$

where  $|\mathbf{v} - \mathbf{U}|$  is the relative velocity of the bubble or particle in a liquid with viscosity  $\nu$ , and  $R_b$  its radius. The measurements are done for  $0.1 < Sr < 1$  and  $0.01 < Re < 100$ . The typical bubble sizes are  $R_b \approx 1$  mm.

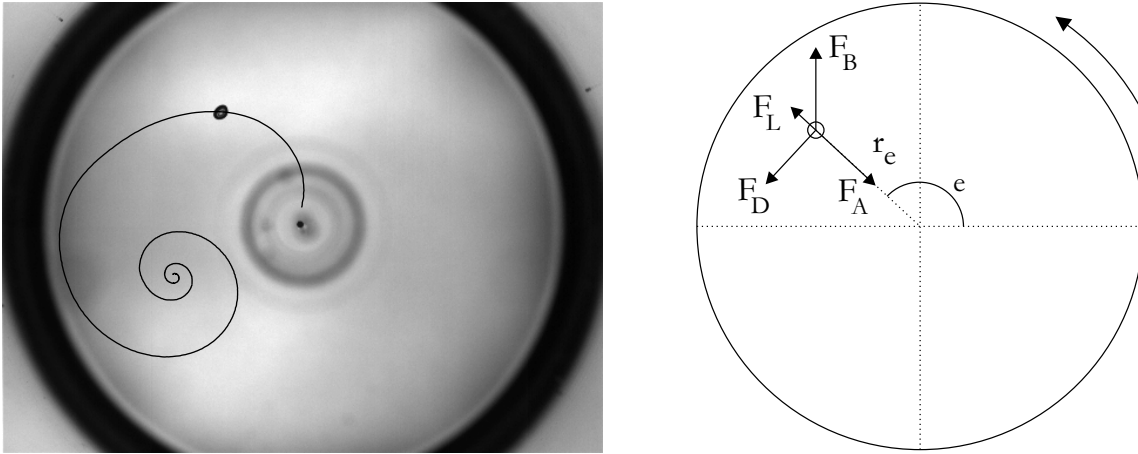
For observation, one digital camera is aligned with the rotation axis of the cylinder, another one orthogonal to it. The gas bubbles are injected into the cylinder at some initial position  $z_0$  with a needle. After injection, the needle is removed. The flow field in the cylinder and the wake of the particle or bubble is visualized with dye. Additionally, the liquid is seeded with polystyrene particles. A laser light sheet allows for visualization of the velocity field in a plane perpendicular to the axis of rotation. The motion of bubbles and particles are recorded at frames rate up to 500 Hz. Particle tracking is used to reconstruct the trajectories.



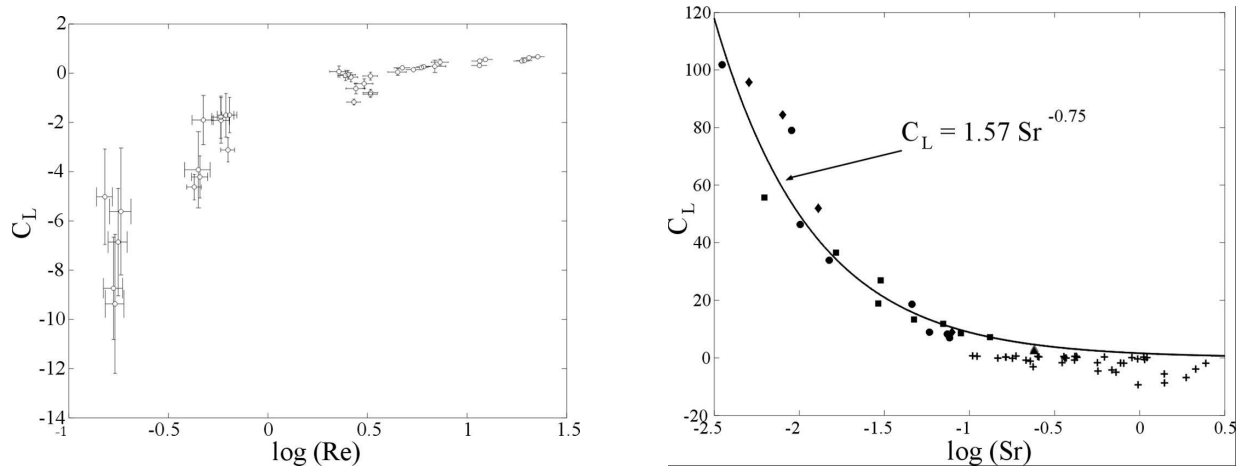
**Figure 1.** Experimental setup.  $L=500$  mm,  $D=100$  mm (a) Side view. (b) Axial view with cylindrical coordinates  $\{r, z, \varphi\}$ . Digital camera aligned along the  $z$ -axis and perpendicular to it.

## RESULTS

Figure 2 shows the transient motion of a bubble after being inserted near the axis of rotation. This dynamics can qualitatively be described by an equation of motion taking into account drag, added mass, buoyancy and lift force [1, 2]. In agreement with experimental observation, spiralling and cycloidal motion are found numerically. Asymptotically, the bubble moves towards its equilibrium position  $\{r_e, \varphi_e; z_0\}$ , from which the lift and drag coefficients  $C_L$  and  $C_D$  can be deduced. We find an increased drag in accordance with numerical experiments in linear shear flow [3]. The dependence of  $C_L$  on the Reynolds number  $Re$  is shown in Fig. 3. Negative lift coefficients are calculated for  $0.1 < Re < 3$ , whereas  $C_L$  converges to values  $0.75 < Re < 1$  for large  $Re$  (see [4]). The dependence of the lift coefficient on the Strouhal number  $Sr$  is shown in Fig. 3. Our results are in good agreement with the data of Sridhar & Katz [5] and Naciri [1]. The empirical model suggested by Sridhar & Katz seems to hold as well, at least for the systematic trends. Finally, translational motion of bubbles parallel to the axis of rotation is observed for  $5 < R_b < 10$  mm and  $15 < \omega <$



**Figure 2.** *Left:* Axial view on the trajectory of a bubble ( $R_b = 2$  mm) spiralling towards its equilibrium position. The inner wall of the cylinder is visible as a dark ring. *Right:* Bubble equilibrium position at  $\{r_e, \varphi_e\}$  due to balance of buoyancy  $F_B$ , drag  $F_D$ , added mass  $F_A$ , and lift  $F_L$ .



**Figure 3.** *Left:* Lift vs. Reynolds for the measurements gly5-gly8. *Right:* Lift vs. Strouhal: gly5-gly8 results (+), Sridhar & Katz' data (■:  $20 < Re < 30$ , ●:  $50 < Re < 70$ , ◆:  $65 < Re < 80$ ), Naciri's data (▲, taken from Sridhar & Katz [5]). Superposed is the empirical model suggested by Sridhar & Katz.

$40 \text{ rad s}^{-1}$ . In this case the Reynolds number is  $Re \approx 140$ . A tentative explanation of the phenomenon is given in terms of bubble–wake interaction.

## References

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