

NATURAL CONVECTION OF PSEUDOPLASTIC FLUIDS INVOLVING SOLID SUSPENSIONS IN WATER

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Summary The numerical experiment of the transient natural convection in two-dimensional cavity with non-Newtonian fluid is presented. In non-Newtonian viscosity models it is assumed that the laminar viscosity can be expressed as a continuous function of the strain rate. The present work is based on the power law model, which is the most popular non-Newtonian viscosity models. The results of numerical experiments of natural convection in two-dimensional cavity with Newtonian or non-Newtonian are presented.

PROBLEM FORMULATION

Many fluids including plastic melts and suspensions and solutions, display a more complicated, nonlinear relationship between the applied stress and their observed strain rate. Such fluids are called non-Newtonian fluids. The power law model is the most popular of non-Newtonian viscosity models.

The transient natural convection in two-dimensional rectangular cavity with power law model of non-Newtonian fluid is considered. A fluid layer is subjected to the gravity and heated from below. A steady thermal field with a horizontal gradient ΔT is applied and induces a steady longitudinal flow that loses its stability when ΔT becomes larger than a critical value. Then, as in the case of vertical convection, for ΔT increasing, several bifurcations occur and a transition to chaos is foreseen for ΔT approaching infinity.

The fluid motion can be described by the dimensionless Navier-Stokes equations for a viscous incompressible non-isothermal fluid:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + \nabla p - \nabla \cdot \mathbf{S} - g \frac{Ra}{Pr} \mathbf{T} \mathbf{k} = \mathbf{0}$$

$$\nabla \cdot \mathbf{v} = \mathbf{0}$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T - \frac{1}{Pr} \nabla^2 T = 0$$

where \mathbf{v} , T , p are, respectively, the velocity, temperature and pressure fields and ρ is the density, g is gravitational acceleration, β is the coefficient of thermal expansion and $\kappa = \frac{k}{\rho c}$ is the conductivity, c is the specific heat at constant volume.

Moreover,

$$\mathbf{S} = \mu(\dot{\gamma}) \mathbf{G}$$

is the extra stress tensor and

$$\mathbf{G} = \nabla \mathbf{v} + (\nabla \mathbf{v})^T$$

is the rate of strain tensor and $\mu(\dot{\gamma})$ is the shear-rate dependent viscosity where $\dot{\gamma}$ is defined as $\dot{\gamma} = \sqrt{\frac{1}{2} II}$, and II is the second invariant of the rate of strain tensor.

In the power law model (also known as the Ostwald-de Waele, or two-parameter model) it is assumed that the laminar viscosity can be expressed as a continuous function of the strain rate

$$\mu(\dot{\gamma}) = A \dot{\gamma}^{B-1}$$

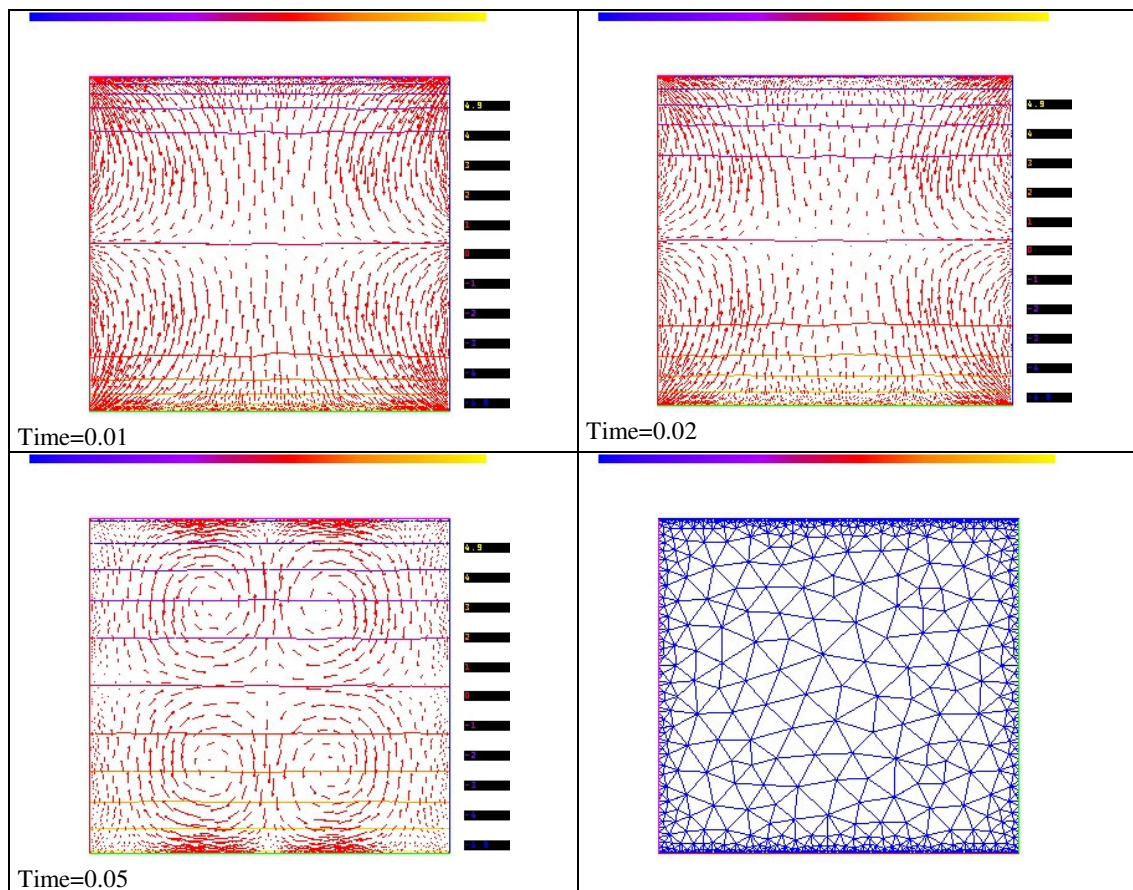
where B is dimensionless constant called the power-law index, and A is an empirical coefficient known as the consistency factor, which serves as an index of how viscous the fluid is. Note that for $B=1$ power law model reduces to Newton's law with $A = \mu$. Thus the deviation on B from unity indicates the degree of non-Newtonian behaviour of the fluid. Specifically, when $b < 1$, the fluid is said to be pseudoplastic and the viscosity is found to decrease with increasing strain. On the other hand, when $B > 1$, the fluid is said to be dilatant and the viscosity increases with increasing strain. Dilatant fluids are generally much less common than pseudoplastic ones. 4% paper pulp in water ($A = 20.0$, $B = 0.575$), 54.3% cement rock in water ($A = 2.51$, $B = 0.153$) are examples of pseudoplastic fluids involving solid suspensions in water.

NUMERICAL RESULTS

The numerical experiment has been performed for Newtonian fluid and two types of non-Newtonian fluids (described above).

The augmented Lagrangian method has been implemented to solving the transient natural convection of fluid in two-dimension square cavity. A fluid layer is subjected to the gravity and heated from below.

The numerical experiment has been performed for the Newtonian fluid with parameters: $Ra=1.e4$; $Pr=1.0$; $Pen=20.0$. The cavity is a unit square. The timestep used in calculations is $\Delta t=0.01$. The bottom wall of the cavity is heated with constant dimensionless temperature $+1$, and the top wall is cooled with -1 .



During each timestep, 3 iterations of the solution of the discretised momentum equation were done in order to obtain a converged velocity field.

The results of non-Newtonian fluid tests are included in full form of presented paper.

References

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