THERMAL BUOYANCY CONVECTION IN SYSTEMS WITH DEFORMABLE INTERFACES

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Summary Thermal convection in a two-layer system with deformable interface heated from below is investigated. The generalized Boussinesq approximation is applied. Long-wave and cellular disturbances are investigated in the framework of linear stability theory. Non-linear equation describing the long-wave disturbances with large amplitude is derived and investigated. Numerical simulation of supercritical convective regimes is conducted.

The paper deals with the investigation of thermal buoyancy convection in a two-layer system with deformable interface. It is well known that interface deformability assumption is incompatible with the conventional Boussinesq approximation. Formally it is related to the fact that Rayleigh number, which is the governing parameter in the Boussinesq theory, can be represented as the product of Galilee number Ga and Boussinesq parameter, characterizing relative density variations due to the temperature inhomogeneities. The Boussinesq approximation uses the limit transition, when Galilee number tends to infinity and Boussinesq parameter tends to zero whereas Rayleigh number remains finite. However, when the Galilee number increases, the deformations of fluid interface tend to zero. At the same time there are situations where deformations of interface could be important. One an example is the case of interface between the fluids with close densities. In this case effective description is possible in the framework of generalized Boussinesq approach, in which not only the Boussinesq parameter but also the relative density difference is assumed to be small. In the present work, in the framework of this model the buoyancy convection of a system of two fluids filling the infinite horizontal layer heated from below is considered. The problem allows the solutions which correspond to the conductive state with motionless fluids and horizontal interface.

The stability of this state is investigated analytically and numerically. For small long-wave disturbances analytical investigation is carried out by the expansion into the wave number series. The stability to the disturbances with finite wave-length in the case of thermally conductive layer boundaries is determined by fundamental partial solution system construction. The substitution of the solutions into the boundary conditions allows to obtain the characteristic equation for the disturbances growth rate.

The calculations show that if the density difference of two fluids is large enough (it is convenient to characterize the density difference by the dimensionless parameter *G*, which has the meaning of the Galilee number defined through the density difference) the interface does not practically deform and the threshold of instability coincides with the well known value for the case of non-deformable interface. With *G* decrease the role of interface deformations grows and the critical Rayleigh number corresponding to the most dangerous disturbances increases, i.e. the stability of the system is also raising. However, at small enough values of *G* the crisis of the conductive state does not correspond to the cellular disturbances, but to the long-wave ones and the critical Raleigh number dependence on *G* is different. Namely, with *G* decreasing the critical Rayleigh number decreases up to the zero value by linear low.

Under assumption of small amplitudes of the long-wave interface disturbances, the finite-amplitude excitation of convection is found to be realized at small supercriticalities in a wide range of parameters. However, the problem under consideration is specially interesting by the possibility to study analytically the case of strong deformations of interface. The point is that if the fluid densities are extremely close then even weak flow leads to the strong deformations of the interface. In this way it was possible to derive the amplitude equation

$$\frac{\partial\zeta}{\partial t}=\frac{\partial}{\partial x}\left(f(\zeta)\frac{\partial\zeta}{\partial x}\right)-C\frac{\partial}{\partial x}\left(f_G(\zeta)\frac{\partial^3\zeta}{\partial x^3}\right)$$

$$f(\zeta)=Ra\, f_R(\zeta)-Gf_G(\zeta)$$

where ζ is the deviation of interface from horizontal position, C is the capillary parameter, $f_G(\zeta)$ and $f_R(\zeta)$ are complicated functions in general case; these functions turn to zero at $\zeta=\pm 1$. The first function is positive at $|\zeta|<1$, and the second function does not have a definite sign. In the limit case, when the viscosity and thermal expansion coefficients of both fluids are the same, the expression for these functions is simplified:

$$f_R=\frac{[\kappa]\left(1-\zeta^2\right)^2}{240}\frac{[\kappa]\left(4\zeta^4-9\zeta^2-7\right)-10\zeta^3+34\zeta}{\left(2-[\kappa]\zeta\right)^2},\quad f_G=\frac{1}{24}\left(1-\zeta^2\right)^3$$

Here $[\kappa]$ denotes the difference of thermal conductivity coefficients divided by their half-sum.

In the framework of this equation the stability of conductive state is investigated (conductive temperature distribution, planar interface). It is shown that direct or inverse bifurcations are possible depending on the parameter values.

The periodical stationary solutions of amplitude equation are investigated numerically. Solutions, corresponding to two-dimensional convective flows in the system with deformed interface, are discovered. These solutions exist in a rather narrow range of parameters. In the other cases one of the fluids splits into the drops. In the case of heating from below the drops of fluid with higher thermal conductivity surrounded by less thermally conductive fluid are formed. Under heating from above the opposite situation is observed: the drops of less thermally conductive fluid are surrounded by higher thermally conductive fluid.

In the parameters range where the cellular disturbances are more dangerous, the investigation of supercritical convective regimes is made numerically by finite difference method using level-set approach for a concrete pair of fluids.