

SEA CONVECTIVE MOTIONS DRIVEN BY RANDOM BUOYANCY INPUTS

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Summary A model for the dynamics of uncorrelated dense water plumes, perturbed by the Coriolis force, and the conditions for their generation, is presented here. A Lagrangian representation allows their time evolution till the initial collective rotating chimney formation to be obtained. Scale relations are examined, depending on the surface air-sea interaction statistics involved; their dependence on buoyancy fluctuations more than mean values is shown

Introduction- The model

Sudden cooling and evaporative events occur on the sea surface during the winter in several regions, inducing buoyancy instabilities and convective mixing, an important mechanism responsible for deep water formation (DWF). A recent review and references are given in Maxworthy (1997) and in Marshall and Schott (1998). The known experimental observations of these phenomena in an open sea can be summed up in: 1) a preconditioning phase during all the winter; 2) a violent sudden plume very short (2h) formation phase; 3) a plume mixing and rotating formation phase; 4) a chimney baroclinic instability phase until the chimney finally breaks off as the external forcing dies down.

The object of the present work is to discuss the plume formation process in a homogeneous ($N^2 = \frac{g}{\rho_0} \frac{\partial \rho}{\partial z} = 0$), and a slowly stratified ($N^2 \approx 10^{-3} - 10^{-4} \text{ sec}^{-2}$) sea, like it is the MEDOC region or the RED sea, DWF sites; for a sake of simplicity the depth dependence of the vertical stratification and the horizontal stratification are ignored: the extension of the model to this more physical situation will be object of future work. The observation of wind data (Zecchetto et al. 2001) measured (a) on 11 November 1994 at 22:02 (during the initial preconditioning phase), and (b) on 18 March 1997 at 22:08 (during the initial DWF event phase) show transverse winds, symmetric around a central axis, very slowly variable along their motion direction in (a), and transverse winds, but radially symmetric around the center of the doming in (b), when the sea site is baroclinic and slowly rotating ($\omega \ll f$); this radial winds symmetry would mean a strong surface air-sea momentum transfer and a strong wind stress; but the mechanical mixing due to the winds would be so deep that the convective motion generated occurs at $1/\lambda \geq 100m$, where the wind stress is anyway not important.

I examine an initially still sea, perturbed at the surface by sudden transversal dry, cool winds, that appear as a large series of random events, short range space correlated, occurring in a short time period, whose result is equivalent to sudden space and time non homogeneous surface cooling, as suggested by Maxworthy about the 1Km large uncorrelated plumes measured by Schott and Leaman (1991) in the Gulf of Lion. By assuming a correlation between the wind speed and the surface buoyancy flux symmetries, I examine a three-dimensional Navier-Stokes model, in (a) a rectangular and (b) a cylindrical frame; in (a) a very slow dependence on y (the winds direction) of the surface buoyancy flux and of the plumes evolution, while in (b) a very slow dependence on the angular polar variable are assumed. It is so possible for both the cases to simplify the three-dimensional model into a couple of equations (for the transversal vorticity $\nabla_{\perp}^2 \psi$ and the transversal velocity v ((b) u_{θ}), uncoupled for short times ($t < 1/f$); i.e. I identify two time scales such that, for $t < 1/f$, a two-dimensional convective motion is generated, but perturbed (for $t \geq 1/f$) by the Coriolis force, such that the full evolution is the combination between a two-dimensional convective motion and a linear transverse one.

The basic model equations (scaled with the horizontal dimension L , and the Coriolis parameter f) are, in Boussinesq approximation:

$$\frac{D\nabla_{\perp}^2 \psi}{Dt} = s_b + s_v + s_N + \mathcal{E} \frac{\partial^2 \nabla_{\perp}^2 \psi}{\partial z^2} \quad ; \quad \frac{Dv}{Dt} = \frac{\partial \psi}{\partial z} + \mathcal{E} \frac{\partial^2 v}{\partial z^2} \quad ; \quad \frac{Db}{Dt} - \mathcal{N}_b w \left(\frac{b}{g} - 1 \right) = \frac{db_s}{dt}$$

where b is the buoyancy, the viscosity term $\mathcal{E} = \frac{\nu}{L^2 f} \approx 3 \cdot 10^{-7}$ is negligible, $t = ft$, $s_b = -\frac{\partial b}{\partial x}$, $s_v = -\frac{\partial v}{\partial z}$, $s_N = -\mathcal{N}_b \frac{\partial P}{\partial x}$ are point vorticity sources; s_b is the primary vorticity source, an antisymmetric x function depending on the horizontal buoyancy gradient produced by the air-sea interaction: $\frac{db_s}{dt} = \frac{db_s(x,t)}{dt} e^{-\lambda z} e^{-y^2/2}$ ($\lambda \approx 1/100m^{-1}$) is the external input, a symmetric x function, random in x and in t ; it is thought off as essentially concentrated near the central axis, and periodically near a set of axes rightly spaced out: a critical distance between them and a suitable decaying range a for each of them are being identified for plumes generation; a stochastic model of a set of short space correlated buoyancy inputs is created such as to have this average periodicity; s_N is a countervorticity source due to stratification (P is the scaled pressure); s_v is a correction term to the vorticity generation due to Coriolis effects: it may be split off into $s_v^b + s_v^N$, working, on long time scales ($t \geq O(1)$), on the horizontal velocities $u(\psi(s_b))$ and $u(\psi(s_N))$ respectively; moreover it is $u = -\frac{\partial \psi}{\partial z}$, $w = \frac{\partial \psi}{\partial x}$; (b) a similar set of equations is given in cylindrical symmetry, with b_s not dependent on θ , $u_r = -\frac{\partial r \psi}{r \partial z}$, $w = \frac{\partial r \psi}{r \partial r}$

The method

The Boussinesq approximation allows the solution of the vorticity equation to be considered a single Dirichlet problem ($\psi = 0$ on the boundary), solvable by image charge method, by using a transformation to Lagrangian variables $\xi = x - \int_0^\tau u(\xi, \eta, \tau') d\tau'$, $\eta = z - \int_0^\tau w(\xi, \eta, \tau') d\tau'$, $\tau = t$, such that: $\frac{D}{Dt} = \frac{\partial}{\partial \tau}$, $\frac{\partial}{\partial x} = a_{11} \frac{\partial}{\partial \xi} + a_{12} \frac{\partial}{\partial \eta}$, $\frac{\partial}{\partial z} = a_{21} \frac{\partial}{\partial \xi} + a_{22} \frac{\partial}{\partial \eta}$; so: $\frac{D\psi}{Dt} = \frac{\partial \psi}{\partial \tau} = \sum_{k=0}^{\frac{2R_0}{na}} \int_0^{\frac{na}{2}} dx' \int_0^h dz_0 s(x', z_0) g_k(x, z, x', z_0) = \sum_{k=0}^{\frac{R_0}{na}} \int dq(x', z_0) g_k(x, z, x', z_0)$ where na is the plumes periodicity, $g_0 = \log \frac{r_2 r_3}{r_1 r_4}$, ($r_i = ((x \pm x')^2 + (z \pm z_0)^2)^{1/2}$), $g_{k \neq 0} = \log \frac{r_{2k}^+ r_{4k}^-}{r_{2k}^- r_{4k}^+}$, $r_{ik} = ((x \pm x' - nka)^2 + (z \pm z_0)^2)^{1/2}$ and the antisymmetric far dipole depending on $r_{ik} = ((x \pm x' + nka)^2 + (z \pm z_0)^2)^{1/2}$ has been neglected. It is known that $g_k \rightarrow \frac{x' z_0}{(x'^2 + z_0^2)^2} |x|z$ for $x, z \rightarrow 0$, ($x = x - nka$) i.e g_k is hyperbolic near each of the axes; moreover: $b(x, z, t) = b(\xi + \int^\tau u(\xi, \eta, \tau') d\tau', \eta + \int^\tau w(\xi, \eta, \tau') d\tau', \tau) + b_s(x, z, t) + \int^\tau (\frac{b}{g} - 1) \mathcal{N}_b w d\tau'$; $\rho(x, z, t) = \rho(\xi + \int^\tau u(\xi, \eta, \tau') d\tau', \eta + \int^\tau w(\xi, \eta, \tau') d\tau', \tau) + \rho_s(x, z, t)$ and, for $t \geq O(1)$, $v(x, z, t) = \int^\tau u(\xi, \eta, \tau') d\tau' + v(\xi + \int^\tau u(\xi, \eta, \tau') d\tau', \eta + \int^\tau w(\xi, \eta, \tau') d\tau', \tau)$. The non linear problem can be linearized if we note that the envelope of all the closed streamfunctions generated by each dq is essentially summed up near the axes where the buoyancy is really mostly concentrated, i.e where an hyperbolic approximation is good, so that $a_{11} = a_{22} = 1$, $a_{12} = a_{21} = 0$ all the time. The consequence is that the buoyancy is dragged along the streamfunction produced by the space integrated vorticity sources situated in 'centers' ($P_k \equiv (\Delta_i^k(t), \zeta_i^k(t))$) such that $\frac{q_i^k}{2} = \int^{\zeta_i^k} dq_i^k(x=0, z)$; $\frac{q_i^k}{2} = \int^{\Delta_i^k} dq_i^k(\zeta, x)$, following the equation $\frac{d^2 \zeta_{qb}^k}{dt^2} = \frac{\partial^2 \psi_k}{\partial \tau \partial x} \Big|_{z=\zeta_{qb}^k}$, with the initial condition: $\zeta_{qb}^k(t=0) = \frac{1}{\lambda} \log 2$, $\zeta_{qN}^k(t=0) = \frac{b}{2}$, $\Delta_{qb}^k(t=0) = \Delta_{qN}^k(t=0) = a \sqrt{2(\log 2 - \log(1 + 2e^{-n^2/8}))}$, soluble for long times, such that $\Delta_i \ll \zeta_i$.

Results-one plume

1) In order to have a sinking plume it is necessary a steep buoyancy gradient, i.e. a has to be $\ll 1$, otherwise only a very slow, not sinking, flow perturbation is visible. 2) A scale invariance with λ is obtained. A single $\delta(t)$ external input, such as $b_s(x) = b_a e^{-\frac{x^2}{2a}}$, generates a single sinking plume, following the law $\zeta(t) = (C < B_0 > t^3)^{\frac{1}{2}}$ (B_0 is the surface buoyancy flux); a numerical calculation verifies this Marshall scale law with $C \approx (3.2 \pm 1.2) \lambda^{-2}$, got for $t \approx 0.3 \gg 1.4 \cdot 10^{-2}$. A stochastic time set of such external inputs, with random b_a , generates a single sinking plume following a higher law power ($t^{5/2}$), or the same, depending on the probability $1/\tau$ of each random event; these 'scaling' laws depend on the mean value B_0 or on its time fluctuations ($< B_0^2 > - < B_0 >^2$) $^{1/2}$, depending on the air-sea interaction statistics. A surface buoyancy produced by a slowly time continuous variable summed to a stochastic set of such events leads, for large values of τ , to small amplitude variance oscillations, whose frequency is dependent on time. A constant surface buoyancy flux leads to Marshall scale laws. 3) An analysis of the internal randomness leads to intermittent plumes, but internal randomness is anyway hidden by the external randomness. 4) The stratification has a perturbative decreasing in time effect on the homogeneous sea vorticity dynamics, depending to the external statistics. 5) For $t \approx O(1)$, the Coriolis effect generates a transverse shear (antisymmetric around $x((b)r) = \sqrt{3}z$): the buoyancy, mostly trapped in the hyperbolic region, is driven along the $y((b)\theta)$ direction; but the slow dependence on $y((b)\theta)$ allows each surface $y = c((b)\theta = c)$ to have the same behaviour, so that the buoyancy evolution is not affected directly in average. The vorticity source s_v lets the perturbation to be horizontally propagated; in a stratified sea for long times this propagation is such as to change the sinking evolution law.

Results-more plumes

1) The sinking plume laws depend on the maximum values of the buoyancy and on the steepness of the gradient, not on its mean values. Statistically they depend on buoyancy space fluctuations. 2) A critical distance $l_c = 2\sqrt{2}a$ is identified. For $l \geq 6a$ a set of independent 'one plume' motions like before, for $2\sqrt{2}a < r < 6a$ a set of independent plumes, with the same time power laws as before, (C depending on the buoyancy gradient), are identified. 3) A short space correlation range between a set of surface buoyancy inputs is identified, such as to have this kind of periodicity. 4) For $t = O(1)$ the collective effect of the transversal shear ((b)of the rotation) is a final rotation on the boundary ((b)a collective chimney rotation). 5) In a stratified sea the before told propagation lets the single plumes to interact in time.

References

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