BIFURCATIONS OF STEADY THERMO-CAPILLARY FLOWS OF A BINARY MIXTURE

Batishev V.A.

Rostov State University, Ul. Zorge 5, 344090 Rostov-na-Donu, Russia

Summary. The paper is focused on thermo-capillar flows of a binary mixture in a narrow horizontal layer. We find several exact solutions of the governing system consisting of the Navier-Stokes, heat and diffusion equations. For these solutions we examine the branching of secondary regimes. Using numerical and asymptotic procedures we identify those values of parameters for which the bifurcation point is detached of the main regime and a reconnection of secondary flows occurs.

BASIC FLOWS.

We study the branching of unsteady self-similar axis-symmetric thermo-capillar flows of the binary mixture in a narrow horizontal layer. The flows are induced by temperature and concentration gradients on the free boundary. The governing system consists of Navier-Stokes, heat and diffusion equations. We let the temperature and concentration (or their fluxes) be given on the rigid lower boundary, and we suppose heat and concentration fluxes to vanish at the free upper boundary. We employ boundary-layer approximation for this boundary-value problem. We study self-similar regimes whose velocity, concentration and temperature fields can be expressed via four functions F, G, T, C satisfying to the following ODE boundary value problem:

$$\begin{aligned}
&F^{(4)} = -h^2(2FF''' + 1.5F'' + 0.5\eta F''' + 2GG') \\
&G'' = h^2(2F'G - 2FG' - G - 0.5\eta G') \\
&T'' = h^2 Pr(2F'T - 2FT' - 1.5T - 0.5\eta T') \\
&c'' = h^2 Pd(2F'c - 2Fc' - 1.5c - 0.5\eta c') \\
&F(0) = F'(0) = G(0) = 0,\ T(0) = \gamma,\ c(0) = \tau \\
&F''(1) + hT(1) - hc(1) = 0,\ F(1) = c'(1) = T'(1) = G'(1) = 0
\end{aligned} \tag{1}$$

Here Pr denotes Prandtl number, Pd denotes Shmidt number, h denotes layer thickness, and γ, τ stay for temperature and concentration gradients. If $G = 0$ then the corresponding flow has zero azimuthal velocity. Such the flows are considered as basic flows. They have been constructed numerically. Several parameter domains in which the solution is not unique have been identified.

BRANCHING OF THE SOLUTIONS.

We show that for certain parameter values the secondary flows with nonzero azimuthal velocity branch from the basic flow. To identify the bifurcation points we examine the eigenvalues $h = H(\gamma, \tau, \mathrm{Pr}, \mathrm{Pd})$ of the linearization of the problem (1) on fixed basic flow. This problem has been solved numerically. Both simple and multiple eigenvalues have been observed. Several bifurcation curves on the plane (h, γ) were constructed. Some parameter domains turned out to be free from bifurcations. At the same time, there exists such parameter domains, in which two or more bifurcation points have been observed.

Equation of branching.

We examine the branching of a basic flow. Generally, the equation of branching has the form $b(h, \beta) = 0$, where b is certain function of h, β, and β has to be determined. Expanding b into the power series nearby the point $h = H$, $\beta = 0$ and making use of Newton's diagram we obtain the branching equation in the form

$$(h - H)b_h + 0.5\beta^2 b_{\beta\beta} + \cdots = 0 \tag{2}$$

Here b_h, $b_{\beta\beta}$ denote partial derivatives of the function b in h and β. The coefficients of the branching equations have been found numerically for various values of the parameters. When $b_h \neq 0$ and $b_{\beta\beta} \neq 0$ the branching equation has two roots. This means that the bifurcation creates two secondary regimes with nonzero azimuthal velocities. They differ only in direction of rotation. The secondary regimes have been constructed asymptotically in the neighborhoods of the bifurcation points. Outside of these neighborhoods they were extended numerically.

SEPARATION OF BIFURCATION POINT.

We consider the case $b_h = 0$ for some $h = H$ and $\gamma = \Gamma$. In this case the eigenvalue is double. The branching equation includes only the second order terms (and terms of higher orders). It can be written in the form

$$(h - H)^2 b_{hh} + \beta^2 b_{\beta\beta} + \cdots = 0 \tag{3}$$

The coefficients of the equation (3) have been determined numerically. Here there are two possibilities: (i) $b_h \cdot b_{\beta\beta} > 0$, (ii) $b_h \cdot b_{\beta\beta} < 0$. In the case (i) the equation (3) has no roots and there is no branching at the point $h = H$. However, in certain neighborhood of the point $h = H$, $\gamma = \Gamma$ there are two bifurcation points, which are connected by a branch of secondary regimes. When h tends to H, and γ tends to Γ these bifurcation points tends one to another. They coincide in the limit and the secondary regimes disappear.

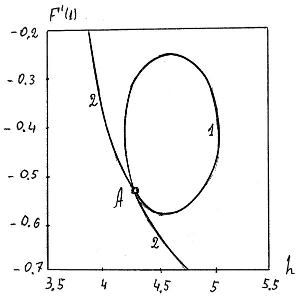

Fig. 1

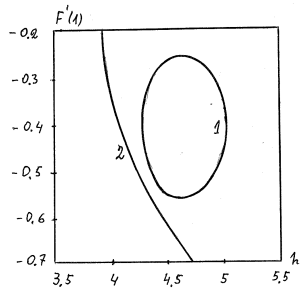

Fig. 2

In the case (ii) the branching equation has four roots. There are two roots when $h < H$ and there are two roots when $h > H$. This means that a two-side bifurcation occurs when $h = H$ and $\gamma = \Gamma$. Such the case is presented on the Fig. 1. The curve 1 corresponds to the main regime, and curve 2 corresponds to the secondary regime. The values of radial velocity on the free boundary are presented on the ordinate axis. The point A denotes the bifurcation point. When $\gamma > \Gamma$ there are two close bifurcation points. They tends to the point A, when h tends to H, and γ tends to Γ, and they coincide with A when $h = H$, $\gamma = \Gamma$. Further decreasing of γ leads to disappearance of the bifurcation point. Here the separation of the bifurcation point and secondary regimes from the basic flow happens. This phenomenon corresponds to the Fig. 2. We note the reconnection of branches of seconday regimes at the point $h = H$, $\gamma = \Gamma$. The secondary regimes have been constructed asymptotically in the neighborhood of this point of separation. Outside of this neighborhood they were extended numerically.

Acknowledgments. This research is supported by Russian Foundation for Basic Research (RFBR), grant (02–01–00226).