INVESTIGATION OF WENO SCHEMES FOR 3D UNSTRUCTURED GRIDS.

Jerzy Majewski*, Robert Wieteska*
*Warsaw University Of Technology, Institute of Aeronautics and Applied Mechanics,
Nowowiejska 24, 00-665 Warsaw, Poland

Summary The paper presents a comparison of WENO (Weighted Essentially Non Oscillatory) reconstruction applied within two common approaches to Finite Volume Method (FVM), i.e. Cell Centered (CC) and Vertex Centered (VC) method, which are used for simulation of compressible inviscid flows. The CC and VC methods were subject to comparison in order to assess their quality with regards to accuracy of the scheme and to the computational cost. The CC method was found producing more accurate results than VC method but it was obtained at higher computational cost. The detailed analysis will be presented for 3D supersonic flow in a channel and for the Onera M6 wing.

METHOD OVERVIEW

The paper presents an algorithm used for simulation of 3D inviscid compressible flows on unstructured grids with Finite Volume Method (FVM).
Flow field obtained with Finite Volume Method is represented by averaged values assigned to each control volume. The fluxes are computed at control volume boundaries and then it is necessary to provide a mechanism for reconstruction of the flow function inside a control volume. The accuracy of the algorithm determines the overall order of the method.
Our approach is using Weighted Essentially Non Oscillatory (WENO) scheme for reconstruction. Main advantage of this scheme lies in possibility to keep high order also in the vicinity of strong discontinuities (shock waves) without introducing oscillations. It is also possible to use this approach for different kind of grids, e.g., structured, unstructured and also grids consisting of mixed type elements. The only information required is an information about neighborhood of a given cell.
The disadvantage of the method is a fact that stencils needed for reconstruction can create quite complicate patterns.

In general the algorithm can be presented in the following way:

- Define for a control volume Ω_h m stencils $S_1, S_2, \cdots, S_m$ consisting of the neighboring control volumes.
- Create a function P_i which approximates solution inside Ω_h for each stencil S_i.
- Calculate oscillation indicator o_i for each function P_i.
- Calculate weights for each P_i using oscillation indicator.
- Find global reconstruction function for a control volume Ω_h as a weighted average of P_i.

CELL CENTERED AND VERTEX CENTERED APPROACHES

Two codes using FVM with WENO reconstruction were developed. The difference between them lies in treatment of control volume. The first code with Cell Centered approach is treating each grid cell as a control volume. The second one with Vertex Centered approach (known also as Cell Vertex) is using a dual mesh and then the control volumes are built around vertices. Since number of cells in typical 3D unstructured tetrahedral grid is around 6 times greater than number of vertices, the Cell Centered approach has to store 6 times more variables (which are defined per control volume) than VC approach. It is clear that for the same grid the Cell Centered method will consume much more memory than Vertex Centered one but we expect at the same time more accurate results since there are more control volumes.
Both codes were using first order reconstruction which should lead to second order accuracy in space. In future development the higher order (second, third) reconstruction is planned.

RESULTS

The detailed analysis of the performance of two discretization approaches will be presented for two 3D cases:

- supersonic flow inside the wedge channel (see Fig. 1).
- transonic flow past Onera M6 wing (see Fig. 2).

CONCLUSIONS

WENO reconstruction gives high quality results for both approaches. As expected Cell Centered method gives more accurate results at higher cost than Vertex Centered one.

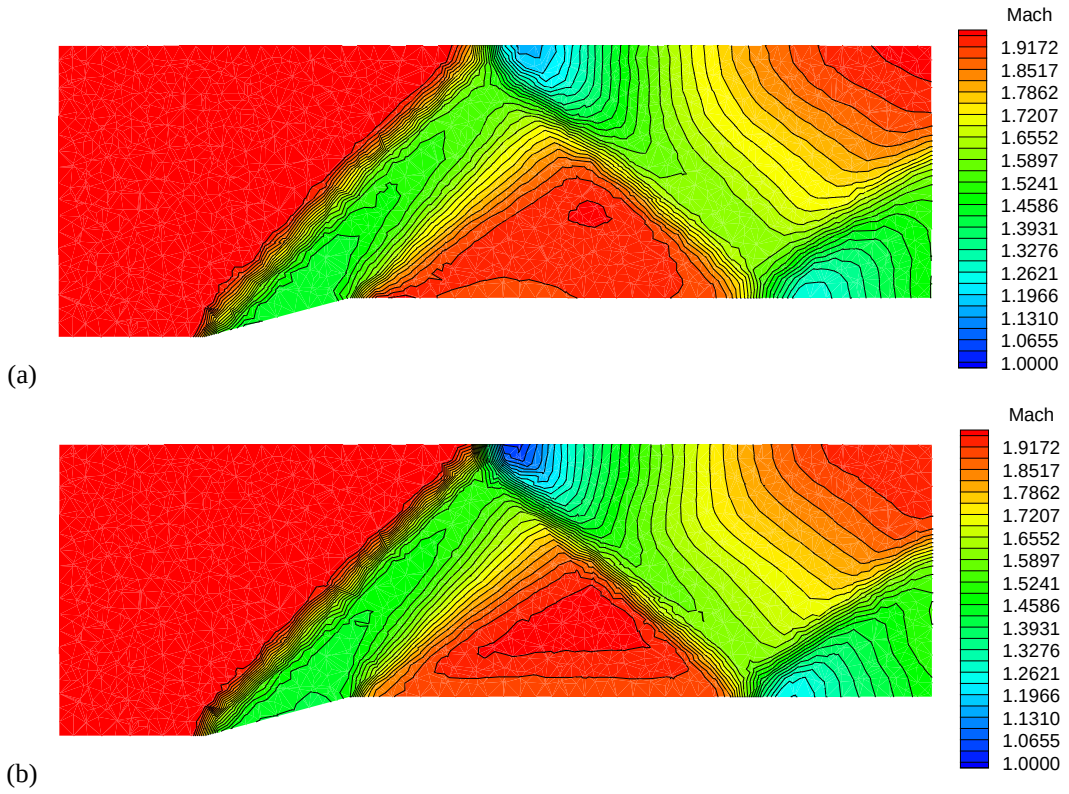

Figure 1. Supersonic flow inside a wedge channel ($Ma_{inlet} = 2.0$).
a) Mach number field calculated with Vertex Centered method (middle cut).
b) Mach number field calculated with Cell Centered method (middle cut).

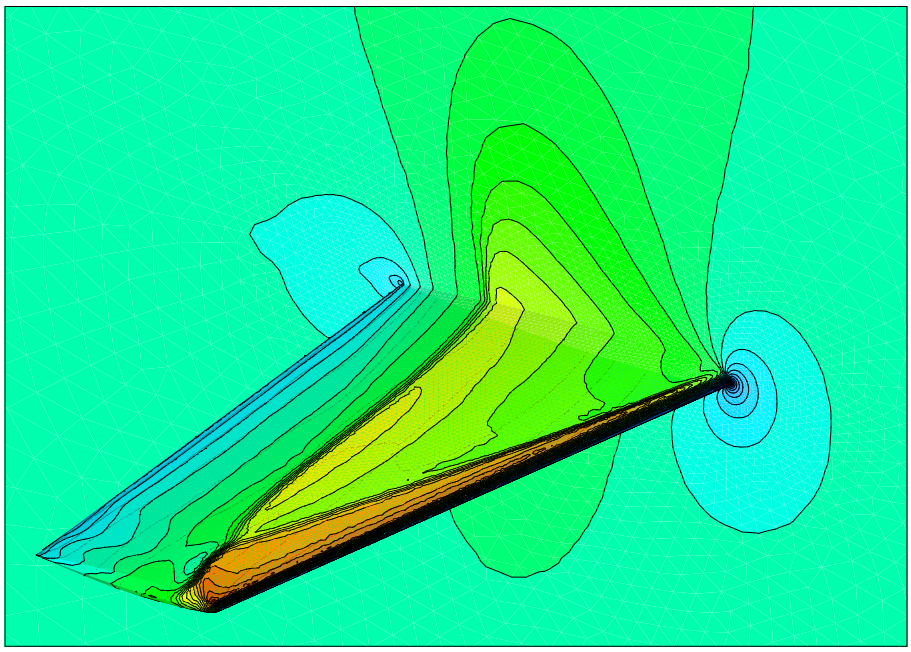

Figure 2. Transonic flow past a M6 Onera wing ($Ma_{\inf} = 0.8$).

References

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