A PRESSURE-CORRECTION METHOD FOR ALL MACH NUMBERS

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Summary We present a collocated finite-volume-based pressure-correction method, for all speed flows of a general fluid. Over the whole Mach number range, the algorithm shows an excellent efficiency and accuracy. Mach-uniform accuracy is obtained by applying the Advection Upstream Splitting Method (AUSM+) for the flux definitions. Mach-uniform efficiency is obtained by treating the convective phenomena and the acoustic/thermodynamic phenomena separately: a velocity predictor from the momentum equations, and a coupled solution of the continuity and energy equation for pressure and temperature corrections.

INTRODUCTION

We present a collocated finite-volume-based pressure-correction method, for all speed flows of a general fluid. Over the whole Mach number range, the algorithm shows an excellent efficiency and accuracy.

MACH-UNIFORM ACCURACY

Mach-uniform accuracy is obtained by applying the Advection Upstream Splitting Method (AUSM+) [1] for the flux definitions. We do not take over the Rhie-Chow interpolation from the incompressible pressure-correction method since it causes an extreme smearing of shocks in high Mach number flows [2]. A pressure dissipation term is added to prevent pressure-velocity decoupling at low Mach numbers. Furthermore, a preconditioned speed of sound is introduced to scale the fluxes properly when the Mach number diminishes [3].

MACH-UNIFORM EFFICIENCY

To obtain Mach-uniform efficiency, the stifness problem has to be remedied. A breakdown of convergence at low Mach numbers occurs if the stability of the scheme imposes an acoustic CFL-limit. This limit is removed if the terms which hold the acoustic information are treated implicitly. Analyzing the Euler equations reveals that the acoustic information is found in specific terms of the momentum and energy equation. The algorithm first determines a predictor value for velocity from the momentum equations. Then corrections are introduced into the implicit terms, which are the acoustic terms in the momentum and energy equations. For the momentum equations this results in a relation between velocity and pressure corrections. In the energy equation, the internal energy ρe, (ρ density, e specific internal energy) is expanded as a function of pressure and temperature, according to the equation of state for a general fluid. Thus, the energy equation contains both pressure and temperature corrections. A second equation containing these two kind of corrections is derived from the continuity equation. The two equations containing both pressure as well as temperature corrections, are solved in a coupled way.
Valid for a general fluid, it is instructive to see how the algorithm behaves in some special cases:

- If density is constant, the continuity equation becomes a pure pressure-correction equation. The energy equation becomes a convection-diffusion equation for temperature corrections. The algorithm therefore reduces to the classical incompressible pressure-correction method [4].
- If an ideal gas is considered, ρe depends only on pressure. In absence of heat transfer, the energy equation then contains only pressure corrections. Thus, a coupled solution with the continuity equation is no longer neccessary: a pressure-correction equation based on the energy equation can be used to update pressure. This is consistent with the low Mach number perturbation analysis for an ideal gas [5]. Figure 1 shows the convergence plots for computations of a one dimensional nozzle flow at a throat Mach number of 0.001. There is no acoustic CFL-limit and the Mach-uniform algorithm shows an excellent convergence rate. Figure 1 also shows the convergence plots for a compressible pressure-correction algorithm that uses the continuity equation to construct the pressure-correction equation. The computations could only be made stable under a severe underrelaxation, and the convergence rate at this low Mach number is very bad. It clearly suffers from the stiffness problem and therefore doesn't reach Mach-uniform efficiency. However, several examples of pressure correction methods based on the continuity equation are found in the literature.
  If heat transfer is present, temperature appears into the conductive terms of the energy equation. To escape a diffusive Neumann limit, these terms have to be treated implicitly. This implies that temperature corrections are introduced there. Therefore, a coupled solution with the continuity equation is needed once again.

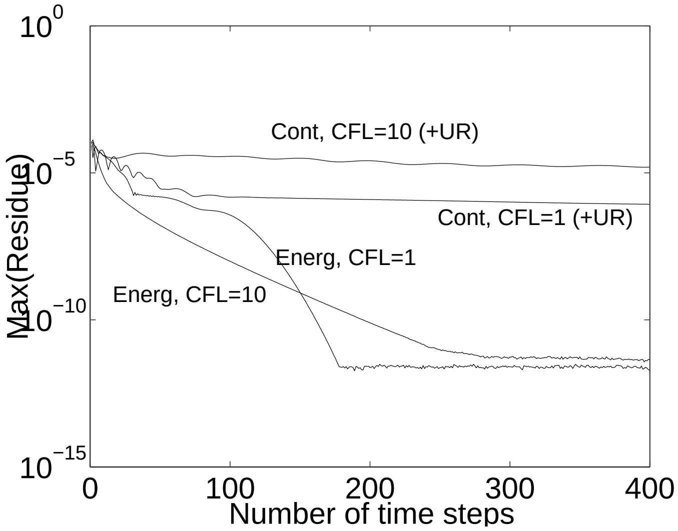

Figure 1. Subsonic nozzle flow, $M_{throat} = 0.001$. Convergence plot for computations at convective CFL numbers of 1 and 10. *Energ*: Mach-uniform algorithm. *Cont*: equivalent algorithm based on the continuity equation (computation with underrelaxation (UR))

CONCLUSION

The described algorithm, with a predictor step from the momentum equation, and a coupled solution of the energy and continuity equation, is the most general form. The essential idea is that convective phenomena and acoustic/thermodynamic phenomena are treated seperately. Doing so, only the appropriate terms are treated implicitly, contrary to fully implicit coupled methods. The described algorithm finds its place in between a fully segregated and a fully coupled approach. It allows an efficient and accurate flow computation of any kind of fluid, at any speed.

References

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