CALCULATION OF VORTICAL STRUCTURE EVOLUTION USING COMBINED DISCRETE SINGULARITY AND BOUNDARY ELEMENT METHOD

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__Summary__ Combined discrete singularity and boundary element method, which is generalization of discrete vortex method and boundary element method, is proposed. The proposed method is applied to the problem of vortical structure evolution. The proposed approach is illustrated by examples of numerical calculations.

Consider a plane flow problem formulated in terms stream function – vorticity and written in integral form

$$\chi(x_0,y_0)\Psi(x_0,y_0)=\oint_\Gamma P(x,y,x_0,y_0)\frac{\partial\Psi}{\partial n}dS-$$
$$-\oint_\Gamma \Psi\frac{\partial P}{\partial n}(x,y,x_0,y_0)dS+\iint_D \omega(x,y)P(x,y,x_0,y_0)dxdy. \quad (1)$$

where $\chi(x_0,y_0)=\begin{cases}0,(x_0,y_0)\notin D,(x_0,y_0)\notin\Gamma,\\ 1/2,(x_0,y_0)\in\Gamma,\\ 1,(x_0,y_0)\in D,\end{cases}$ $\quad P=\frac{1}{2\pi}\ln\left(\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2}}\right)$,

D is flow domain, Γ is boundary of domain D, P is fundamental solution of Laplace equation or corresponding Green function. Assume that vorticity field is approximated as

$$\omega(x,y)=\sum_{i=1}^{M}\omega_i\delta(x-x_i,y-y_i)\,, \quad (2)$$

here M is number of free discrete vortices, ω_i are their intensities. Thus

$$\chi(x_0,y_0)\Psi(x_0,y_0)=\oint_\Gamma P(x,y,x_0,y_0)\frac{\partial\Psi}{\partial n}dS-$$
$$-\oint_\Gamma \Psi\frac{\partial P}{\partial n}(x,y,x_0,y_0)dS+\sum_{i=1}^{M}\omega_i P(x_i,y_i,x_0,y_0), \quad (3)$$

Sometimes, the representation (3) is not convenient for calculation, especially if discrete vortices interact on small distance. In that case velocity field becomes singular and to avoid this difficulty the discrete vortices must be replaced by some other model. Consider, as an example, the vortical dipole creation in vortical sheet after thin body under small attack angle. Assume that the vortical sheet is formed by two sheets separated from lower and upper sides of the airfoil.

$$\Psi(x,y)=\omega_1P(x,y,x_1,y_1)+\omega_2P(x,y,x_2,y_2)$$
$$\Psi(x,y)=(\omega_1+\omega_2)P(x,y,x_0,y_0)+(\omega_1h_1-\omega_2h_2)\frac{\partial P}{\partial h}+\cdots$$

where the point (x_0,y_0) is position of new construction, approximating vortices pair, h_1 is a distance between the points (x_1,y_1) and (x_0,y_0), h_2 - between the points (x_2,y_2) and (x_0,y_0). There are doublet of $\omega_1h_1-\omega_2h_2$, and vortex of $\omega_1+\omega_2$.

Thus there are dipoles in the flow domain. In this case equation (3) must be replaced by following equation

$$\chi(x_0,y_0)\Psi(x_0,y_0)=\oint_\Gamma P(x,y,x_0,y_0)\frac{\partial\Psi}{\partial n}dS-$$
$$-\oint_\Gamma \Psi\frac{\partial P}{\partial n}(x,y,x_0,y_0)dS+\sum_{i=1}^{M}\omega_i P(x_i,y_i,x_0,y_0)+\sum_{i=1}^{K}d_iP_1(x_i,y_i,x_0,y_0,n_{xi},n_{yi}), \quad (4)$$

here P_1 is dipole influence function, K is number of the dipoles, d_i are their intensities, n_{xi}, n_{yi} is normal vector to dipole axes. It is evident, that $P_1 = \frac{\partial P}{\partial n}$.

As a result of dipole action the considered vortical sheet can be destroyed.

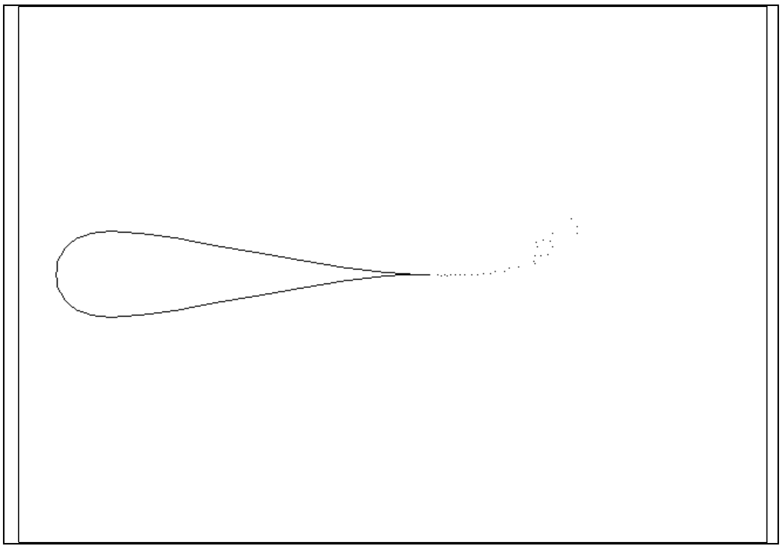

Figure 1. Calculation of starting stage of the vortex sheet after trailing edge of symmetric airfoil under small attack angle. Proposed combination of discrete vortices dipoles is used in calculation. The dots correspond to discrete vortices locations.

The following boundary element method algorithm with integration along real boundary is used (formulation for Laplace equation with respect unknown u)

$$0 = \sum_{k=1}^{N} \int_{\Gamma_k} \varphi_{1k}(\alpha_1^k, \alpha_2^k, \alpha_3^k, ..., \alpha_n^k, x(t), y(t)) \times \frac{\partial G}{\partial n}(x_i, y_i, x(t), y(t)) h_k(t)dt - \\ - \sum_{k=1}^{N} \int_{\Gamma_k} \varphi_{2k}(\alpha_1^k, \alpha_2^k, \alpha_3^k, ..., x(t), y(t)) G(x_i, y_i, x(t), y(t)) h_k(t)dt, \quad (4)$$

where $\varphi_{1k}, \varphi_{2k}$ are base functions, approximating u and $\frac{\partial u}{\partial n}$, correspondingly; $\alpha_1^k, \alpha_2^k, ..., \alpha_n^k$ are the set of arbitrary values to be determined in the solution process of system (4); $x_{i,}$ y_i are the set of the control points, using in the system (4) solution; $h_k(t)$ is shape-function of curve, the form of which depends on method of boundary describing. Using of algorithm (4) gives an opportunity to calculate velocity field near the boundary very accurately as contrasted to by conventional method. The example of vortical structure interaction with solid body is shown on figure 2.

Three mode of vortical structure evolution are observed in numerical investigation. The first one is unstable mode. The second one is absolutely stable mode, which corresponds to high symmetry level. And the last mode is “stable as a cloud”, that is vortical structure saves its geometrical form, doesn’t save size and relative discrete vortex positions. As a result of occasional disturbance accumulation absolutely stable structure can transform to “stable as a cloud”, and “stable as a cloud” can transform to unstable structure. It is shown that the dipole can leads to destruction of the most stable vortical structure, for example, vortical rings (see figures 3 and 4).

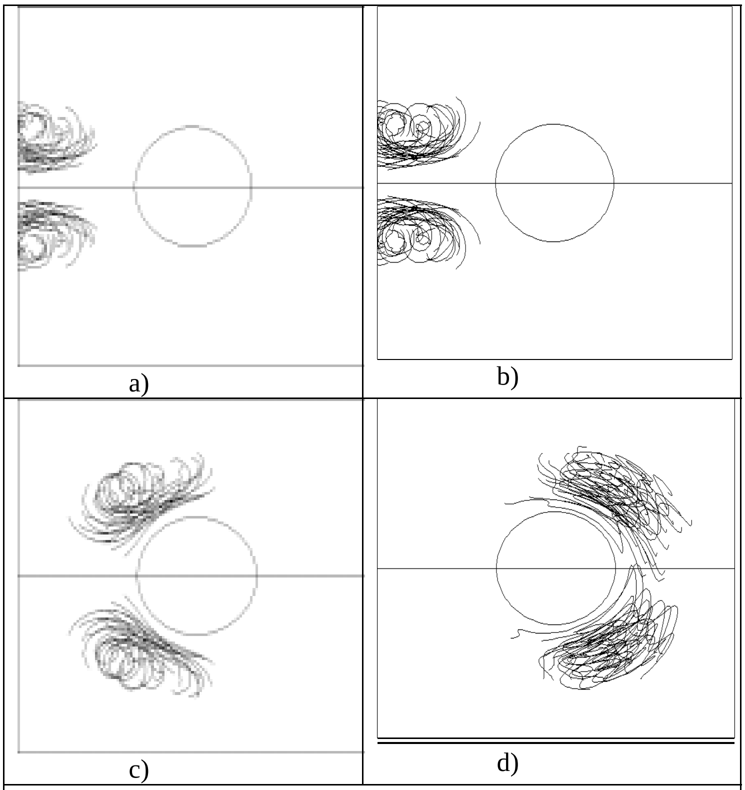

Figure 2. Interaction of vortical structures of different signs with circular cylinder.

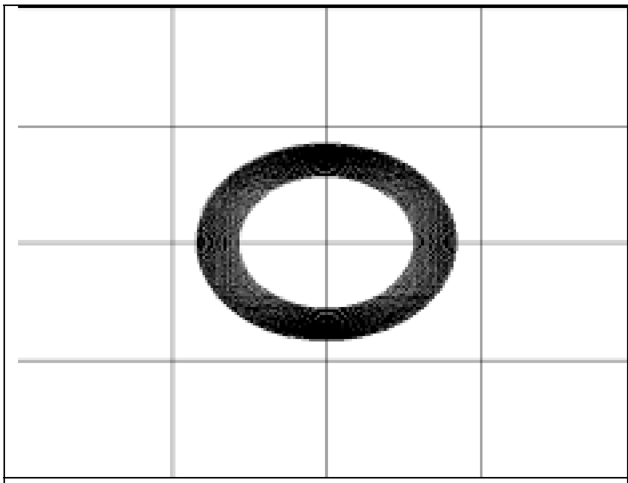

Figure 3. Vortical ring is the most stable vortical structure

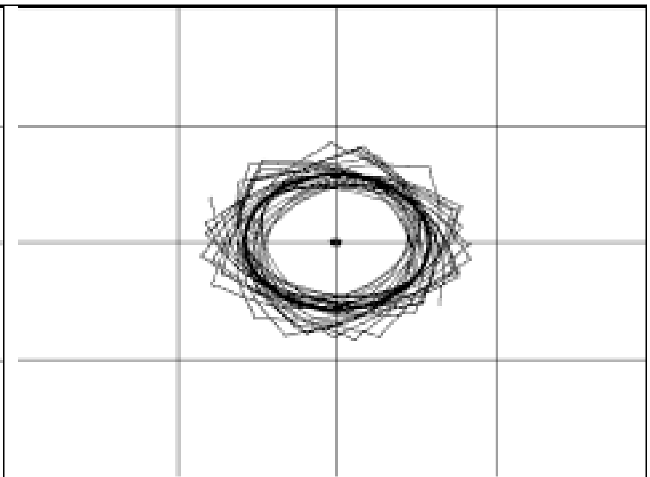

Figure 4. Destruction of vortical ring by central dipole