THE MACH REFLECTION OF WEAK SHOCKS

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Summary We present numerical solutions of weak shock Mach reflections that contain a remarkably complex sequence of supersonic patches, triple points, and expansion fans immediately behind the leading triple point. This structure resolves the von Neumann triple point paradox of weak shock Mach reflection.

During the second world war, von Neumann carried out an extensive study of shock reflection [5]. He suggested a number of criteria for the transition from regular to Mach reflection, and compared his theoretical results with observations. There was generally excellent agreement for strong shocks, but for weak shocks serious discrepancies were found. In particular, a pattern closely resembling simple Mach reflection is observed for weak shocks, but no standard triple point configuration is compatible with the jump relations across shocks and contact discontinuities. This discrepancy was called the 'von Neumann triple point paradox' by Birkhoff [1]. Many different resolutions of this paradox have been suggested over the years [3].

In this paper, we present numerical solutions of initial and boundary value problems for the unsteady and steady transonic small disturbance equations that provide an asymptotic description of weak shock Mach reflection. These solutions contain a sequence of supersonic patches, shocks, expansion fans, and triple points in a tiny region behind the leading triple point. We conjecture that this sequence is infinite for an inviscid weak shock Mach reflection. At each triple point, there is an additional expansion fan, thus resolving the apparent conflict with von Neumann's theoretical arguments. Furthermore, an infinite sequence of shrinking supersonic patches resolves theoretical difficulties connected with the transition from supersonic to subsonic flow at the rear of the supersonic region.

The existence of a supersonic patch and an expansion fan at the triple point of a weak shock Mach reflection was proposed by Guderley [2], although he did not give evidence that this is what actually occurs, nor did he suggest that there is, in fact, a sequence of supersonic patches and triple points. Previous numerical solutions of weak shock Mach reflections with supersonic patches behind the triple point were obtained in [4, 7, 8], but none of these solutions were sufficiently resolved to show the true structure of the solution.

An asymptotic shock reflection problem for the unsteady transonic small disturbance equations, which have the normalized form

$$u_t + \left(\frac{1}{2}u^2\right)_x + v_y = 0, \qquad u_y - v_x = 0, \tag{1}$$

together with appropriate initial and boundary conditions, may be derived from the compressible Euler equations by the method of matched asymptotic expansions [4]. A numerical solution of this problem, from [6], using a highly refined grid near the triple point is shown in Figure 1. There is a sequence of supersonic patches, shocks, expansion fans, and triple points in a tiny region immediately behind the leading triple point. The change from supersonic to subsonic flow at the rear of the region by means of a subsonic triple point appears to be impossible because no expansion fan can then occur, and a continuous change across a sonic line is unlikely because it would result in an apparently overdetermined boundary value problem for a partial differential equation of mixed-type (the hyperbolic region would be enclosed by a sonic line that lacks a 'gap'). This argument, together with the numerical results, suggests that the sequence of supersonic patches is infinite.

This phenomenon of repeating supersonic patches is likely to be widespread, and we will also show recent numerical results of similar steady Mach reflections that occur in transonic flows.

References

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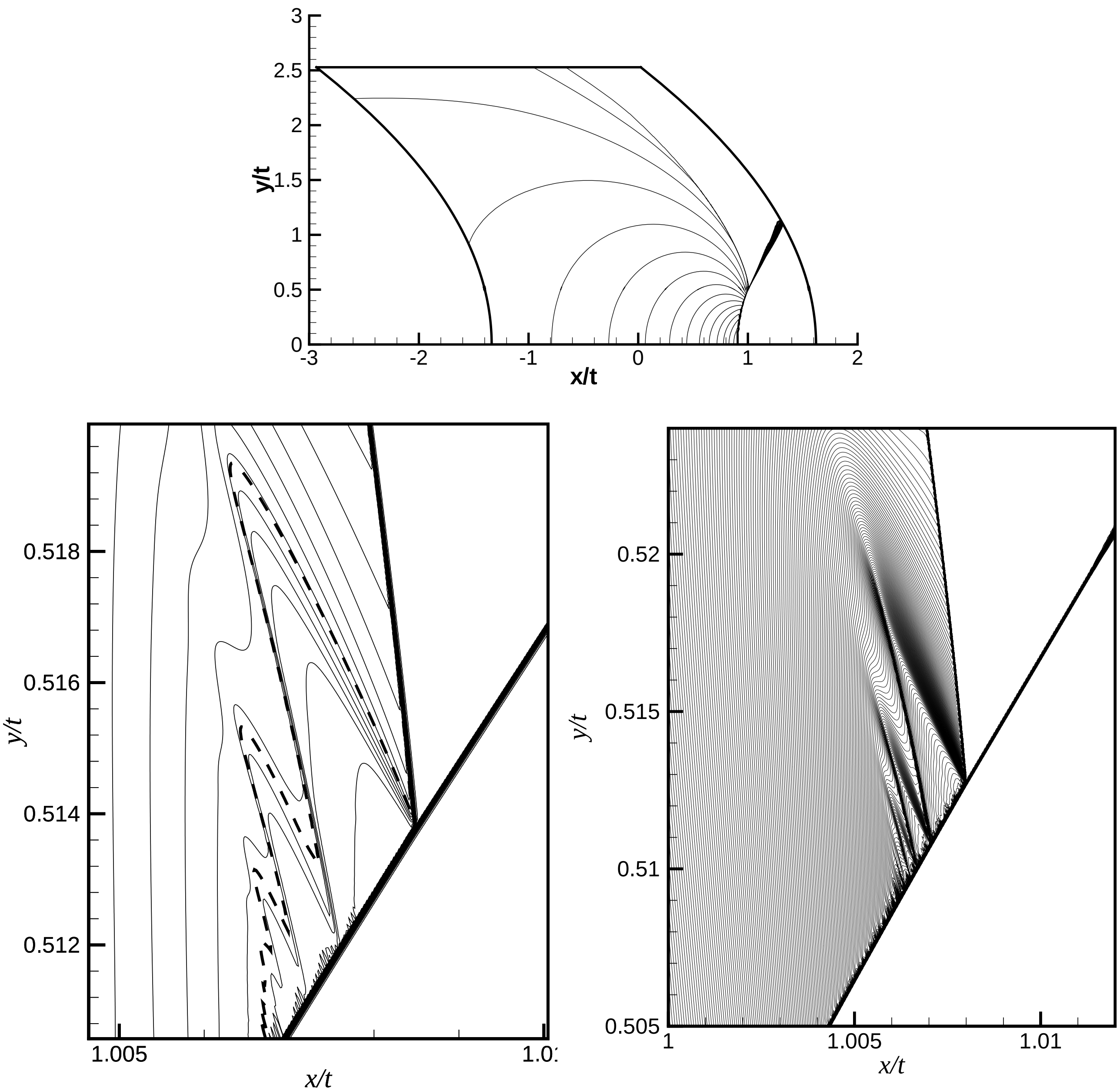

Figure 1. Plots of u-contours in a self-similar Mach reflection solution of the unsteady transonic small disturbance equation (1). The top picture shows the global solution as a function of self-similar coordinates $(x/t, y/t)$. The lower picture shows the local structure of the solution in a tiny region near the triple point, with a sequence of shocks, fans, triple points, and supersonic patches immediately behind the leading triple point. The dashed line in the lower left picture is the numerically computed location of the sonic line where the self-similar equations change type.