

## FERROHYDRODYNAMIC JETS, SHEET FLOWS, AND INSTABILITIES

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**Summary** We investigate the behavior of initially circular cross-section ferrofluid jets impacting a solid circular surface (10 mm diameter) to create an expanding sheet flow in the presence of a magnetic field. The expanding sheet changes from circular with no magnetic field to elliptical with a magnetic field transverse to the jet axis, and remains circular but with a decreased radius for a magnetic field parallel to the jet axis [1].

### DEFORMATION OF THE JET AND SHEET WITH MAGNETIC FIELD PERPENDICULAR TO JET AXIS

Two solenoidal coils apply a horizontal magnetic field transverse to the vertical jet axis which changes the jet cross-section from circular to elliptical, with long axis in the direction of the applied magnetic field. As the transverse magnetic field is increased the expanding sheet also becomes approximately elliptically shaped but with long axis perpendicular to the magnetic field, which is transverse to the long axis of the jet cross-section. This tendency is shown in Figure 1 for oil-based ferrofluid with saturation magnetization  $\mu_0 M_s \approx 387$  Gauss and low-field magnetic permeability  $\mu / \mu_0 \approx 2.6$ . At large magnetic fields, the sheet forms sharp tips and fluid chains emerge from its corners. The prime cause of the change in sheet shape is the influence of the magnetic field on the jet shape. If an initially elliptical non-magnetic jet strikes a circular impactor, the expanding sheet is similarly elliptical but with its long axis also perpendicular to that of the jet.

### SHEET FLOW AND INSTABILITY WITH MAGNETIC FIELD PERPENDICULAR TO INTERFACES

If the applied magnetic field is vertical, and thus perpendicular to the expanding circular sheet, the sheet diameter decreases with increasing magnetic field  $B$ . This tendency is shown in Figure 2 for oil-based ferrofluid. At approximately 500 Gauss magnetic field, the sheet radius decreases by about 40%. A simple Bernoulli analysis including magnetic surface forces predicts the sheet radius to be

$$r = r_T \left[ 1 - \frac{B^2}{\rho U^2} \left( \frac{1}{\mu_0} - \frac{1}{\mu} \right) \right], \quad r_T = \frac{\rho U Q}{4\pi\sigma} \quad (1)$$

where  $r_T$  is the Taylor radius [2],  $\mu$  is the ferrofluid magnetic permeability,  $\mu_0$  is the magnetic permeability of the free space surrounding the sheet,  $\rho$  is the ferrofluid density,  $U$  is the ferrofluid sheet velocity,  $Q$  is the jet flow rate, and  $\sigma$  is the interfacial surface tension [1].

### KELVIN-HELMHOLTZ INSTABILITY

As with the classic case of non-magnetic liquids, the moving sheet upper and lower interfaces are coupled and can become Kelvin-Helmholtz unstable [3, 4]. A planar sheet model, generalized to include a magnetic field  $B$  perpendicular to both interfaces, is formulated to examine the effects of magnetic fields on interfacial stability for a fluid layer of thickness  $\delta$  moving with horizontal velocity  $U$  [1]. Letting  $\xi_1$  be the upwards deflection of the upper interface and  $\xi_2$  be the upwards deflection of the lower interface, a linear stability analysis shows that there are two interfacial modes with  $\xi_1 / \xi_2 = \pm 1$ . For interfacial deflections of the form  $\xi_{1,2} = \text{Re} \left[ \hat{\xi}_{1,2} e^{j(\omega t - kx)} \right]$  the dispersion relation is

$$\frac{\rho_0 \omega^2}{k} + \frac{\rho(\omega - kU)^2}{k} \left( \frac{\coth(k\delta/2)}{\tanh(k\delta/2)} \right) - \sigma k^2 + \frac{k(\mu - \mu_0)^2 B^2 \left[ \cosh k\delta + \frac{\mu}{\mu_0} \sinh k\delta \pm 1 \right]}{\mu \sinh k\delta (\mu^2 + \mu_0^2 + 2\mu\mu_0 \coth k\delta)} = 0 \quad \xi_1 / \xi_2 = \mp 1 \quad (2)$$

where  $\rho_0$  is the density of the surrounding air. Interfacial deflections are unstable if  $\omega^2 < 0$  as then  $\omega$  is imaginary so that interfacial deflections will grow exponentially with time. The  $\xi_1 / \xi_2 = 1$  mode generally has the fastest growth rate of instability. With  $B=0$ , the  $\xi_1 / \xi_2 = 1$  mode becomes unstable at  $k=0$  for Weber number  $We = \rho U^2 \delta / \sigma > 2$ . Since non-zero  $B$  tends to decrease  $\omega^2$ , the magnetic field tends to destabilize the interfacial deflections.

### References

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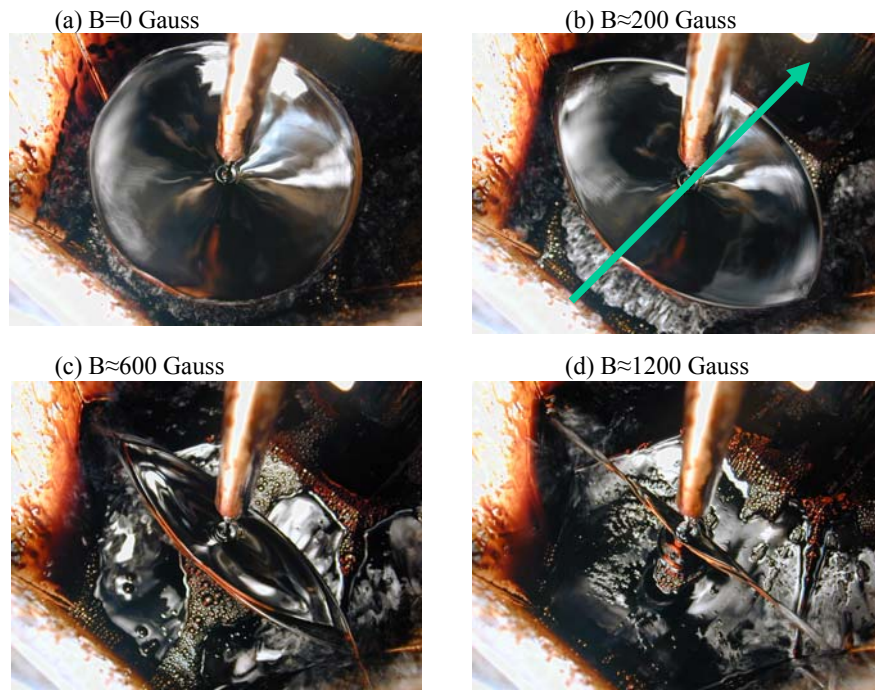


Figure 1 - A vertical oil-based ferrofluid jet, with diameter 2.5 mm, impacts a small circular horizontal plate of 10 mm diameter creating a radially expanding thin sheet flow. (a) In zero magnetic field, a circular jet will create a circular sheet; (b) Application of the magnetic field transverse to the jet, in the direction of the arrow, causes the jet cross-section to elongate in the direction of the applied field while the sheet distorts to an approximately elliptical shape, but with long-axis perpendicular to the applied magnetic field. (c, d) For large magnetic fields, the sheet becomes very thin and is characterized by sharp tips and fluid chains emerging from its corners.

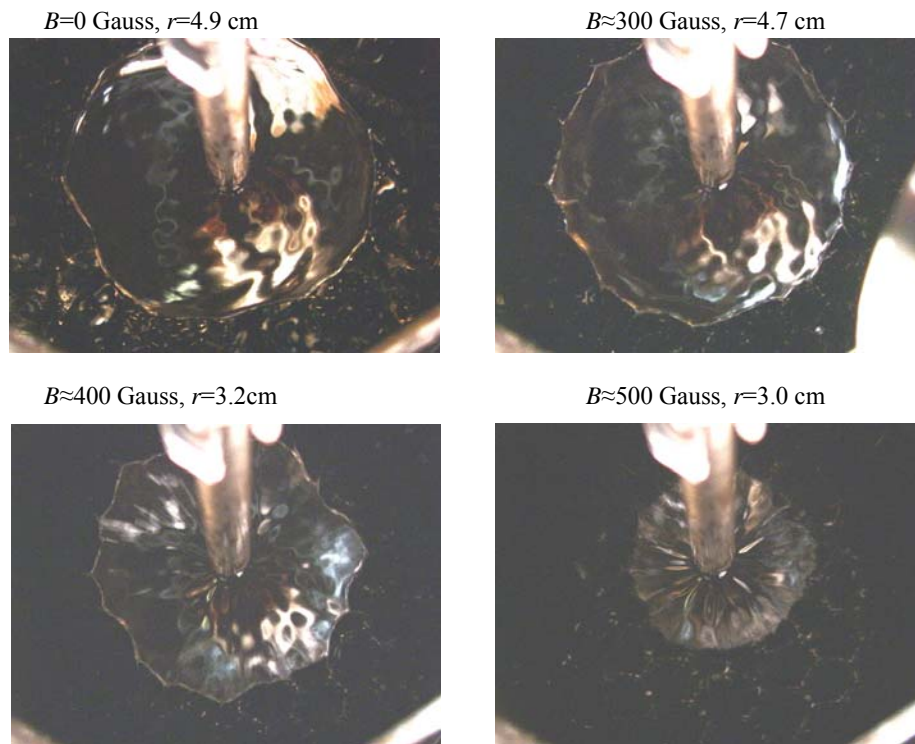


Figure 2 – The expanding oil-based ferrofluid sheet radius  $r$  decreases with increasing perpendicular magnetic field, where  $U \approx 2.3 \text{ m/s}$ ,  $Q \approx 1.2 \times 10^{-5} \text{ m}^3/\text{s}$ ,  $\rho = 1.17 \times 10^3 \text{ kg/m}^3$ ,  $\sigma \approx .026 \text{ N/m}$ , and  $\mu/\mu_0 \approx 2.6$ . The Weber number for the jet of diameter  $d=2.5 \text{ mm}$  is  $We = \rho U^2 d / \sigma \approx 600$ .

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