EFFECTIVE MAGNETOVISCOSITY FOR FERROFLUID PLANAR COUETTE FLOW

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Summary The effective magnetoviscosity and spin velocity for ferrofluid planar Couette flow is calculated for an applied uniform DC magnetic field transverse to a duct axis.

MAGNETIZATION

For low magnetic fields we use Shliomis' first magnetization relaxation equation [1, 2] for magnetization $\overline{M}$ in a magnetic field $\overline{H}$,

$$\frac{\partial \overline{M}}{\partial t} + (\overline{v}\bullet\nabla)\overline{M} - \overline{\omega}\times\overline{M} + \frac{1}{\tau}\left[\overline{M} - \chi_0\overline{H}\right] = 0 \tag{1}$$

where we assume the ferrofluid is incompressible $(\nabla\bullet\overline{v}) = 0$ and at low fields obeys a linear constitutive law in equilibrium with constant magnetic susceptibility χ_0. The time constant τ is the effective magnetization relaxation time related to Brownian and Néel time constants.

For the planar duct shown in Figure 1, the flow velocity $\overline{v}$ and the spin velocity $\overline{\omega}$ are of the form

$$\overline{v} = v_z(x)\overline{i}_z, \quad \overline{\omega} = \omega_y(x)\overline{i}_y \tag{2}$$

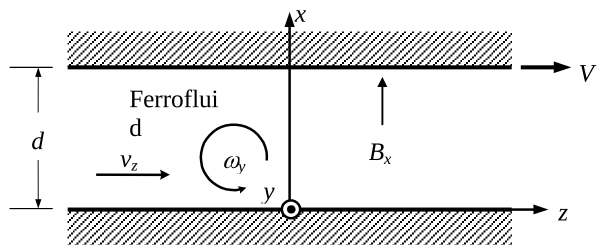

Figure 1 – A planar ferrofluid layer between rigid walls, in planar Couette flow driven by the *x*=*d* surface moving at velocity *V*, is magnetically stressed by a uniform *x* directed DC magnetic field B_x.

For this geometry, all field and flow quantities are independent of *y* and *z* and can only vary with *x*. Gauss's law for the magnetic flux density $\overline{B} = \mu_0(\overline{H} + \overline{M})$ and Ampere's law for the magnetic field $\overline{H}$ with zero current density are

$$\nabla\bullet\overline{B} = 0 \Rightarrow \frac{dB_x}{dx} = 0 \Rightarrow B_x = \text{constant} \tag{3}$$

$$\nabla\times\overline{H} = 0 \Rightarrow \frac{dH_z}{dx} = \frac{dH_y}{dx} = 0 \Rightarrow H_y, H_z = \text{constant=0} \tag{4}$$

At DC, the magnetic fields are of the form

$$\overline{B} = \left[B_x\overline{i}_x + B_z(x)\overline{i}_z\right], \quad \overline{H} = H_x(x)\overline{i}_x \tag{5}$$

Using (2) – (5) in (1) yields the magnetization components

$$M_x = \frac{\chi_0 B_x/\mu_0}{\left[(\omega_y\tau)^2 + 1 + \chi_0\right]} \quad ; \quad M_z = \frac{-\chi_0 B_x\omega_y\tau/\mu_0}{\left[(\omega_y\tau)^2 + 1 + \chi_0\right]} \tag{6}$$

MAGNETIC FORCE AND TORQUE DENSITIES

The magnetic force density is

$$\overline{f} = \mu_0(\overline{M}\bullet\nabla)\overline{H} = \mu_0 M_x\frac{dH_x}{dx}\overline{i}_x = \mu_0 M_x\frac{d}{dx}\left(\frac{B_x}{\mu_0} - M_x\right)\overline{i}_x = -\overline{i}_x\frac{d}{dx}\left(\frac{1}{2}\mu_0 M_x^2\right) \tag{7}$$

There is no *z* directed force to drive the flow. The magnetic torque density is

$$\overline{T} = \mu_0(\overline{M}\times\overline{H}) = \mu_0 M_z H_x\overline{i}_y = \left(M_z B_x - \mu_0 M_x M_z\right)\overline{i}_y = \frac{-\chi_0\omega_y\tau B_x^2\left[\left(\omega_y\tau\right)^2 + 1\right]}{\mu_0\left[\left(\omega_y\tau\right)^2 + 1 + \chi_0\right]^2}\overline{i}_y \tag{8}$$

FLUID FLOW AND SPIN VELOCITIES

For low Reynolds' number flows, so that inertia is negligible, and with the neglect of spin viscosity, pressure gradient, and gravity effects, the flow and spin velocity equations are

$$0 = \overline{f} + 2\zeta\nabla\times\overline{\omega} + \left(\zeta + \eta\right)\nabla^2\overline{v} \tag{9}$$

$$0 = \overline{T} + 2\zeta\left(\nabla\times\overline{v} - 2\overline{\omega}\right) \tag{10}$$

where ζ is the vortex viscosity and η is the dynamic viscosity. Using the solution form of (2), (9) – (10) reduce to

$$\left(\zeta + \eta\right)\frac{d^2 v_z}{dx^2} + 2\zeta\frac{d\omega_y}{dx} = 0 \tag{11}$$

$$T_y - 2\zeta\left(\frac{dv_z}{dx} + 2\omega_y\right) = 0 \tag{12}$$

with solution $v_z(x) = \frac{Vx}{d}, \quad \omega_y = \text{constant}$ (13)

where $v_z(x=0)=0$ and $v_z(x=d)=V$. The constant value of ω_y is found from (8) and (13) in (12) by solving the 5th order equation

$$2\omega_y - \frac{\chi_0 \omega_y \tau \left[\left(\omega_y \tau \right)^2 + 1 \right] B_x^2}{2\mu_0 \zeta \left[\left(\omega_y \tau \right)^2 + 1 + \chi_0 \right]^2} + \frac{V}{d} = 0 \quad (14)$$

The shear stress at the *x=0* and *x=d* interfaces is $T_{zx} = (\eta + \zeta)\frac{dv_z}{dx} + 2\zeta\omega_y = (\eta+\zeta)\frac{V}{d} + 2\zeta\omega_y$ (15)

which has no direct magnetic stress, even if $H_z \neq 0$, because H_z and B_x are continuous across the interfaces. In the absence of magnetic field $(B_x = 0)$, the solutions to (14) and (15) are $\omega_{yo} = -\frac{V}{2d}, \quad T_{zx0} = \eta \frac{V}{d}$ (16)

The change in shear stress due to the magnetic field gives the change in viscosity, $\Delta\eta$, as

$$\Delta T_{zx} = T_{zx} - T_{zx0} = \zeta\left(\frac{V}{d} + 2\omega_y\right) = \Delta\eta \frac{V}{d} \Rightarrow \Delta\eta = \zeta\left(1 + \frac{2\omega_y d}{V}\right) \quad (17)$$

Defining the non-dimensional parameters $r = \Delta\eta/(2\zeta) = \left(\frac{1}{2} + \frac{\omega_y d}{V}\right)$ and $P = \frac{\chi_0 B_x^2 \tau}{\mu_0 (1+\chi_0)^2 4\zeta}$, (14) can be rewritten as the 5th order equation

$$\left(r - \frac{1}{2}\right)^5 + \frac{1}{2}\left(r - \frac{1}{2}\right)^4 + \frac{\left[(1+\chi_0)^2 P + 2(1+\chi_0)\right]}{(V\tau/d)^2}\left(r - \frac{1}{2}\right)^3 + \frac{(1+\chi_0)}{(V\tau/d)^2}\left(r - \frac{1}{2}\right)^2 + \frac{\left[(1+\chi_0)^2 (P+1)\right]}{(V\tau/d)^4}\left(r - \frac{1}{2}\right) + \frac{(1+\chi_0)^2}{2(V\tau/d)^4} = 0 \quad (18)$$

Note that similar analyses by Shliomis [2] and Rosensweig [1] have a 3rd order equation in *r* while we have a 5th order equation. The reason is that their analyses took H_x to be the source field in defining *P*. This is the best variable if the duct filled the gap between pole-faces of area *A*, of an infinitely magnetically permeable magnetic circuit, driven by a current *I* through an *N* turn winding so that $H_x = NI/d$, independent of ferrofluid spin velocity and magnetization. The magnetic flux density, $B_x = \mu_0(H_x + M_x)$, would not be a good variable because M_x depends on ω_y given in (6). If the ferrofluid duct in the magnetic circuit gap includes a surrounding air-gap, then H_x would also depend on M_x. Then it would be better to assume a magnetic flux source λ across the *N* turn winding so that $B_x = \lambda/(NA)$ is independent of ferrofluid magnetization and spin velocity. Rosensweig's analysis [1] has some small errors, but the correct form can be obtained from (18) by taking $\chi_0 \ll 1$, as then $B_x \approx \mu_0 H_x$. In this limit, (18) reduces and factors to

$$\left[r^2 - r + \frac{1}{(V\tau/d)^2} + \frac{1}{4}\right]\left[r^3 - r^2\left[\frac{1}{4} + \frac{1+P}{(V\tau/d)^2}\right] + r - \frac{P}{2(V\tau/d)^2}\right] = 0 \quad (19)$$

The quadratic solution only has complex roots while the cubic equation is the corrected form of Rosensweig's analysis valid for any value of χ_0 [1]. Eq. (18) can be solved for *P* in terms of *r* giving the plot in Figure 2 of $\Delta\eta/\zeta = 2r$ versus *P* for various values of $V\tau/d$ with $\chi_0 \approx 0$ and $\chi_0 = 1.55$, corresponding to EFH1 Ferrotec Corp. oil-based ferrofluid used in our experiments.

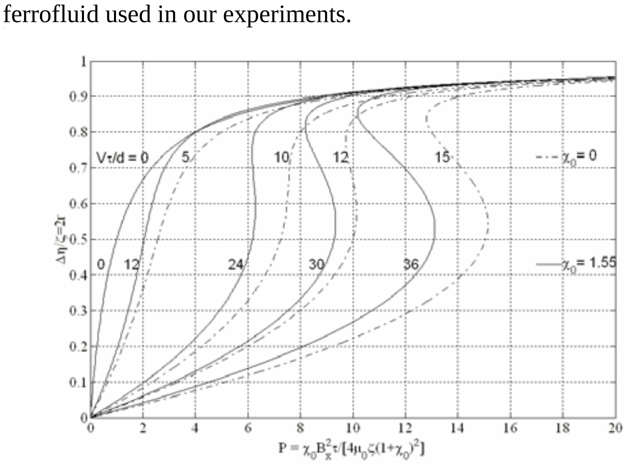

Figure 2 – The solution to (18) for $2r = \Delta\eta/\zeta$ versus $P = \chi_0 B_x^2 \tau/(\mu_0(1+\chi_0)^2 4\zeta)$ for various values of $V\tau/d$ with $\chi_0 \approx 0$ and $\chi_0 = 1.55$. The $\chi_0 \approx 0$ plots reduce to the Shliomis [2] and Rosensweig [1] analyses, with $P = \mu_0 M_x H_x \tau/(4\zeta)$.

Acknowledgment: This research is supported by National Science Foundation Grant# CTS-0084070.

References

[1] Rosensweig R.E: Ferrohydrodynamics. Dover, Mineola, NY 1997.
[2] Shliomis M.I.: *Effective Viscosity of Magnetic Suspension*. Soviet Phys. JETP **34** (6): 1291-94, 1972.