EFFECT OF HORIZONTAL COMPONENT OF THE CORIOLIS FORCE ON PROPAGATION OF NEAR-INERTIAL WAVES IN THE OCEAN

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Summary Oceanic motions of scales small compared to the Earth’s radius are commonly described as if the rotating Earth were locally flat with the horizontal component of the Coriolis force being neglected (the so-called “traditional approximation”). We show that taking into account the horizontal component of the Coriolis force changes dramatically dynamics of near-inertial waves. A new family of sub-inertial waves, which are absent under the “traditional approximation”, is found to play a crucial role: On the “non-traditional” beta-plane near inertial waves are described by the equation

\[ f' \cdot \nabla \psi + N^2 \nabla^2 \psi = 0 \]

where \( f' \) denotes the horizontal Laplacian. We allow the buoyancy frequency \( N \) to depend on \( z \). In the traditional approximation one would take \( f' = 0 \).

For plane monochromatic \( (w = W \exp(i\sigma t)) \) waves travelling in the \( \chi = x \cos \alpha + y \sin \alpha \) direction, we find

\[ (N^2 - \sigma^2 + f_s^2)W_{xx} + 2ff_s W_{xz} - (\sigma^2 - f^2)W_{zz} = 0, \]

where \( f_s = f \sin \alpha \). Equation (2) is the starting point of our study.

On employing the standard boundary conditions at the surface and the bottom the substitution of \( W = \psi(z) \exp i(kx + \delta z) \), with \( \delta = -kf_s/(\sigma^2 - f^2) \) leads to the following BVP for \( \psi \)

\[ \psi'' + k^2 \left[ \frac{N^2(z) - \sigma^2}{\sigma^2 - f^2} + \left( \frac{\sigma f_s}{\sigma^2 - f^2} \right)^2 \right] \psi = 0, \quad \psi(0) = \psi(H) = 0. \]

Note, that from purely mathematical perspective the “non-traditional term” containing \( f_s \), does not pose any difficulty, as it produces merely an additive constant. However it results in the following qualitative effects. In the vicinity of the inertial frequency the solution becomes independent on \( N(z) \) and can be readily found for the \( n \)-th mode

\[ (\sigma^2 - f^2)/\sigma = \pm \frac{f_s H}{2\pi n} k, \quad k > 0 \]

In contrast to the “traditional” boundary-value problem where for all eigenmodes \( \sigma^2 - f^2 > 0 \), we, in addition, have another family of sub-inertial modes with \( \sigma^2 - f^2 < 0 \), which being confined to a narrow \( O((f_s/N)^2) \) frequency range are trapped in the waveguides around minima of \( N(z) \), i.e near surface and bottom of the ocean. In the longwave limit frequencies of the modes of both families tend to \( f \) with finite group velocity \( \sigma/\beta \).

The importance of the new family of sub-inertial waves, which are absent under the “traditional approximation”, becomes apparent if the beta effect or any kind of large scale nonuniformity is taken into account. On the “non-traditional” beta-plane near inertial waves are described by the equation

\[ (N(z)^2 - \sigma^2 + f_s^2)W_{yy} + 2ff_s W_{yz} - (\sigma^2 - f^2 - 2f \beta y)W_{zz} = 0, \quad W(0) = W(H) = 0 \]

which is the starting point of further analysis. Under the “traditional approximation” inertial waves propagating poleward slow down approaching their inertial latitude and then are totally reflected back. On the “non-traditional” beta-plane the picture of wave evolution is entirely different: near inertial waves propagating poleward do not slow down on reaching the inertial latitude, they pass it with finite group mixing.
velocity and propagate further poleward turning into sub-inertial modes trapped within near bottom and near surface wave-guides. On passing the inertial latitudes the horizontal and vertical scales of waves rapidly decrease and tend to zero at a critical latitude, beyond which there is no wave propagation. The critical latitude is located at a distance a few hundred kilometres from the inertial latitude. The picture resembles wave propagating in a wedge. There is no reflection and, thus, inertial waves are absorbed contributing to ocean mixing. Since the main sub-inertial guide is near the bottom, it is expected that the outlined mechanism might be the key one in the so far unexplained mixing in deep ocean.

Figure 2. Behaviour of the characteristic curves $\xi_\pm = const$ for the model $N = const$. The sub-inertial regime is entered via $\xi_+$ (solid line), which connects to $\xi_-$ as it reflects at the critical latitude. The alternation between $\xi_+$ and $\xi_-$ continues upon each reflection at bottom or critical latitude. Inertial and critical latitudes are again indicated by the vertical dotted lines at $y = 0$ and $y = 136$ km. A $\xi_+$-curve that has passed the diagonal dotted line cannot escape anymore from the sub-inertial domain. The dashed line represents an example of an equatorward returning path. The dotted diagonal line representing a $\xi_- = const$ characteristic, marks a borderline: any characteristic $\xi_+ = const$ that reflects at the surface to the left of this curve will return equatorward, as is illustrated by the dashed path; all others will get trapped in the lower-right corner of the sub-inertial domain. It thus appears that most of the poleward propagating energy will get trapped into this corner.