CONTACT-LINE EFFECTS ON CAPILLARY-GRAVITY WAVES

José A. Nicolás*, Javier Hilario**
* *E.T.S.I. Aeronáuticos, Universidad Politécnica de Madrid, Plza. Cardenal Cisneros, 3, 28040 Madrid*
***E.T.S.I. Aeronáuticos, Universidad Politécnica de Madrid, Plza. Cardenal Cisneros, 3, 28040 Madrid*

The damping rate and frequency of capillary-gravity waves in a circular cylinder has proven to be surprisingly difficult to predict quantitatively. The experimental values of damping are usually larger than the theoretical predictions. This difficulty is associated with the fact that several sources contribute to the damping: a) viscous dissipation at the walls of the containers, b) viscous dissipation in the interior fluid, c) viscous dissipation at the free surface due to contamination, and d) dissipation due to contact line dynamics. Of these sources only a) and b) may be calculated from first principles; mechanisms c) and d) must be modeled by phenomenological formulas [1]. The precise theoretical prediction of damping due to a) and b) is essential in order to avoid the indeterminacies associated with sources c) and d). This problem has been recently solved [2], through the incorporation of interior and second order boundary layer dissipation. These calculations showed a satisfactory agreement with the experimental measurements ([1] and [3]) in the case of clean surface and fixed contact line. When the contact line is fixed the mechanism d) is absent. In this case, the effect of contamination is modelled through Marangoni elacticity with insoluble surfactants ([4] and [5]). This model has provided a reasonable agreement with experiments [1] (discrepancies are within 15%), by choosing a finite Marangoni elasticity appropriately [6].

Actually, there are discrepancies between the experimental measurements and the theoretical predictions, only in the cases when the contact line is moving and/or the contact angle is not right.

The effect of the dynamics associated with moving contact lines is modelled through a phenomenological condition suggested by Davis [7] in a different context. We assume that the slope of the contact line is proportional to its velocity. This condition (which includes the two extreme cases of free and pinned end conditions) has been used by several authors ([8] and [1]) in the context of capillary-gravity waves. When the static contact angle is not right, the equilibrium position of the free surface is not flat and the determination of the natural frequencies becomes much more difficult. These effects have been studied experimentally [**?**], however there are not any published theoretical analysis of this problem. The main goal of this work is to cover the lack of theoretical results on these problem. We calculate the frequency and damping rates when the above-mentioned effects are included.

The natural frequency and damping rate of capillary-gravity waves in a circular cylinder with moving contact lines and clean surfaces depend on the following non-dimensional parameters: the inverse gravitational Reynolds number $C = \nu/(gR^3)^{1/2}$ (with $\nu =$ kinematic velocity, $g =$ gravitational acceleration, and $R =$ radius of the cylinder), the Bond number $B = \rho g R^2/\sigma$ (with $\rho =$ density and $\sigma =$ surface tension), the aspect ratio of the cylinder $\Lambda = d/R$ (with $d =$ depth of the cylinder), the static contact angle θ_0, the wetting parameter $H = \lambda/(Rg)^{1/2}$ (with $\lambda =$ phenomenological constant) and the radial and azimuthal wave number of each excited mode. Most experiments on capillary-gravity waves have been made with water or silicone oils and small cylinders ($1\,\text{cm} \leq R \leq 5\,\text{cm}$). In these conditions C and B^{-1} are small. The limit $C \to 0$ (when the contact line is fixed) required to include terms of order C to predict correctly the damping. In the limit $B^{-1} \ll 1$, and where $\theta_0 \neq \pi/2$, there is small static meniscus (of characteristc size $\sqrt{B^{-1}}$) in the vicinity of the contact line. We have studied this effect on the natural frequencies when $C \equiv 0$. We have also analysed the effects of moving contact lines in the case of right contact angle (when there are not static meniscus) and nearly inviscid liquid ($C \ll 1$). Finally, the results are compared with experimental results showing a significant improvement.

References

[1] Henderson D.M. and Miles J.W: Surface-wave damping in a circular cylinder with a fixed contact line. *J. Fluid Mech* **275**:285, 1994.
[2] Martel C., Nicolás J.A. and Vega J.M.: Surface-wave damping in a brimful circular cylinder. *J. Fluid Mech* **360**:213, 1998, see also Corrigendum **373** :379, 1998.
[3] Howell D.R. et al: Measurements of surface-wave damping in a container. *Phys. Fluids* **12**:322, 2000.
[4] Dorrestein R.: General linearized theory of the effect of surface films on water ripples. *Proc. K. Ned. A B* **54**:260, 1951.
[5] Miles J.W.: Surface-wave damping in closed basins. *Proc. R. Soc. Lond. A* **297**:459, 1967.
[6] Nicolás J.A. and Vega J.M.: A note on the effect of surface contamination in water wave damping. *J. Fluid Mech* **410**:367, 2000.
[7] Davis S.H.: Moving contact lines and rivulet instabilities. Part 1. The static rivulet. *J. Fluid Mech* **98**:225, 1980.
[8] Hocking L.M.: The damping of capillary-gravity waves a rigid boundary. *J. Fluid Mech* **179**:253, 1987.
[9] Henderson D. et al: The effects of static contact angles on standing waves. *Phys. Fluids A* **4**:2320, 1992.