PROPAGATION AND INTERACTIONS OF NONLINEAR INTERNAL GRAVITY WAVE BEAMS

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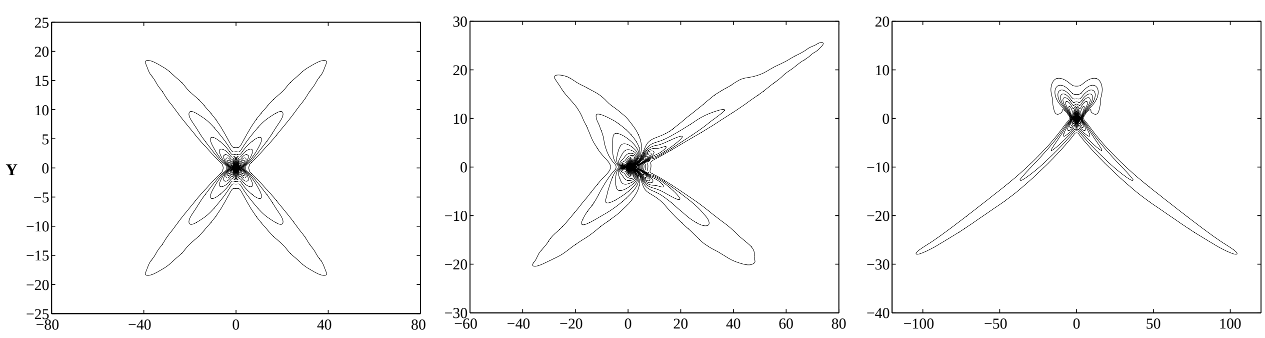
Summary A locally confined source oscillating with frequency below the buoyancy frequency in a stratified fluid generates beam-like wave structures propagating along specific directions. A theoretical study is made of nonlinear effects in such wave beams. Nonlinearity is found to be particularly important in the reflection of a wave beam from a sloping wall and in collisions of two beams propagating along different directions.

PROPAGATION OF NONLINEAR WAVE BEAMS

Based on linear inviscid theory, a two-dimensional source oscillating with frequency ω_0 in a uniformly stratified (constant buoyancy frequency N_0) Boussinesq fluid induces a steady-state wave pattern, also known as St Andrew's Cross, that features four straight wave beams stretching radially outwards from the source at angles $\pm\cos^{-1}(\omega_0/N_0)$ relative to the vertical. Similar wave beams are generated by oscillatory stratified flow over topography. In addition, there is now evidence from numerical simulations and field observations that a significant component of thunderstorm-generated gravity waves in the atmosphere is in the form of beam-like structures. Dewan et al (1998), in particular, were able to link internal wave beams in satellite images of the upper stratosphere to isolated thunderstorms.

All existing theoretical models of the St Andrew's Cross and previous discussions of internal gravity wave beams are based on the linearized equations of motion, valid for small-amplitude disturbances. In the present study, exploiting the fact that uniform plane-wave beams of infinite extent happen to be exact solutions of the nonlinear inviscid equations of motion, we study the propagation of finite-amplitude wave beams taking into account weak viscous and refraction effects. Oblique beams ($\omega_0 < N_0$) are considered first and an amplitude-evolution equation is derived assuming gradual variations along the beam direction. Remarkably, the leading-order nonlinear terms cancel out in this evolution equation and, as a result, the steady-state similarity solution of Thomas $\&$ Stevenson (1972) for linear viscous beams is also valid in the nonlinear régime. Moreover, for the same reason, nonlinear effects are found to be relatively unimportant for nearly-vertically propagating ($\omega_0 \approx N_0$) beams. This explains the success of previous studies interpreting results from fully numerical simulations and field observations on the basis of linear theory. Owing to the fact that the group velocity vanishes when $\omega_0 = N_0$, however, the transient evolution of nearly vertical beams takes place on a slower time scale than that of oblique beams; this is shown to account for the discrepancies between the steady-state similarity solution of Gordon $\&$ Stevenson (1972) and their experimental observations. Based on the present asymptotic theory, we also study the refraction of nearly vertical nonlinear beams in the presence of background shear and variations in the buoyancy frequency. Figure 1(a) shows a wave-beam pattern generated by a locally confined source oscillating with dimensionless frequency $\omega_0 = 1$ in a uniformly stratified Boussinesq flow. Refraction of this wave pattern due to background shear and variations in the buoyancy frequency are shown in figures 1(b) and 1(c), respectively.



(a) (b) (c)

Figure 1. Contour plot of the amplitude of the vertical velocity at time $T = 4$ for wave pattern generated by oscillatory forcing at the origin $x = Y = 0$: (a) uniformly stratified flow with dimensionless buoyancy frequency $N^2 = 1 + 0.04$; (b) uniformly stratified flow with the same buoyancy frequency as in (a) in the presence of background shear $\overline{u} = 2 + 0.1Y$; (c) non-uniformly stratified flow with $N^2 = 1 + 0.01(4 - \frac{3}{4}Y)$ and $\overline{u} = 0$.

REFLECTIONS AND INTERACTIONS

While the effects of nonlinearity are relatively insignificant in the propagation of an isolated finite-amplitude beam, nonlinear effects turn out to be particularly important when two wave beams propagating along different directions interact. We study two examples of such interactions: the reflection of an incident wave beam from a sloping wall and the collision of two beams propagating in different directions.

In the case of reflection from a sloping wall, nonlinear interactions are confined solely in the vicinity of the sloping wall where the incident and reflected beams meet, as each of these beams is an exact nonlinear solution. At higher orders, this interaction region then acts as a source of a mean and higher-harmonic disturbances, and the latter are capable of radiating in the far field, forming additional reflected beams. Depending on the flow geometry, higher-harmonic beams can be found on the opposite side to the vertical than the primary reflected beam (see figure 2). Similarly, when two oblique beams collide, nonlinear interactions in the vicinity of the collision region induce secondary beams with frequencies equal to the sum and difference of those of the colliding beams. The theoretical results are consistent with numerical simulations and experiments.

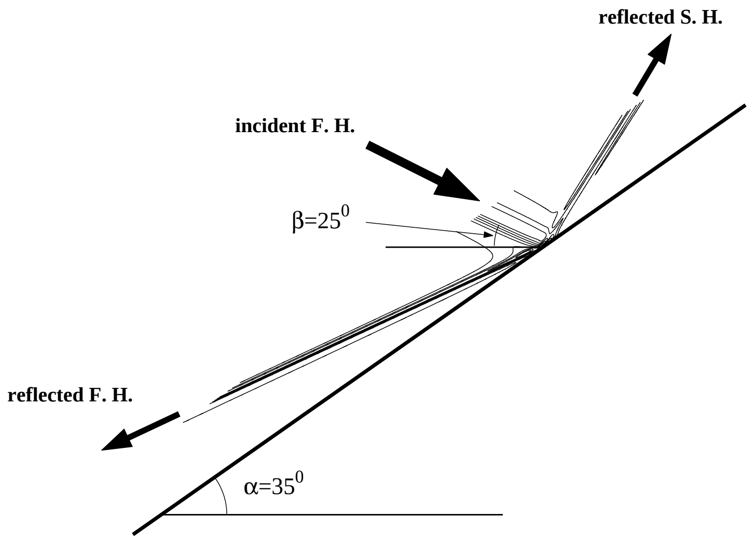


Figure 2. Contour plot of the stream function ψ for incident wave beam of angle $\beta = 25^0$ reflecting from a slope of angle $\alpha = 35^0$. F.H. and S.H. denote first harmonic and second harmonic, respectively.

References

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