THE MODELLING OF THE DYNAMICS OF HAIRPIN VORTEX PACKETS IN WALL TURBULENCE

Vyacheslav V. Meleshko*, Eugene I. Nikiforovich**, Alexandre A. Gourjii**, Ronald J. Adrian***

* *Department of Theoretical and Applied Mechanics, Kiev National Taras Shevchenko University, 01033 Kiev, Ukraine*

** *Institute of Hydromechanics NASU, 03057 Kiev, Ukraine*

*** *Department of Theoretical and Applied Mechanics, University of Illinois at Urbana-Champaign, Urbana, 61801 IL, USA*

Summary The talk addresses the experimental, analytical and numerical modelling of the dynamics of concentrated vortex packets over a rigid smooth plane. To answer the principal question why and how does fluid in outer region of the turbulent boundary layer organize itself into hairpin streamwise vortex packets with low-speed convective velocity we developed the vortex filament model of hierarchy of hairpin packets.

INTRODUCTION

The coherent structure of the turbulent boundary layer has been studied for about fifty years. Based on a combination of analysis and physical insight, Theodorsen in 1952 proposed a simple vortex model as the central element of the turbulence generation process in shear flows. It took the form of a hairpin (or horseshoe)-shaped vertical structure inclined in the direction of mean shear (see Fig. 1). Since that time a large amount of turbulence structure models has been proposed by numerous investigators, see e.g. [1] for a review. These models typically involve similar single or multiple configurations of hairpin vortical structures. Probably the most convincing evidence of the existence of such vertical structures embedded in fully developed turbulent boundary layers has come from recent experimental [2] and computational (e.g.[3, 4]) studies. The idea is to explore the flow of effectively inviscid fluid with embedded vorticity, with topology change allowed upon close encounters of vortical fluid regions.

VORTEX FILAMENT ε-MODEL

We consider thin vortex filaments over a flat rigid plane (with no-penetration conditions) embedded into a shear flow of an effectively inviscid fluid. Virtual mirror filaments maintains no-penetration condition. The induced velocity field is defined by means of the Biot-Savart law:

$$U_x = \frac{1}{4\pi}\int_L \Gamma(x',y',z')\frac{(y-y')dz'-(z-z')dy'}{\left\{(x-x')^2+(y-y')^2+(z-z')^2\right\}^3},$$

$$U_y = \frac{1}{4\pi}\int_L \Gamma(x',y',z')\frac{(z-z')dx'-(x-x')dz'}{\left\{(x-x')^2+(y-y')^2+(z-z')^2\right\}^3},$$

$$U_z = \frac{1}{4\pi}\int_L \Gamma(x',y',z')\frac{(x-x')dy'-(y-y')dx'}{\left\{(x-x')^2+(y-y')^2+(z-z')^2\right\}^3}.$$

The main difference from traditional models is that we consider the strength of the filament dependent on its position. Lagrangian vortex models track fluid elements containing vorticity. The kinematics is governed by geometric relations.

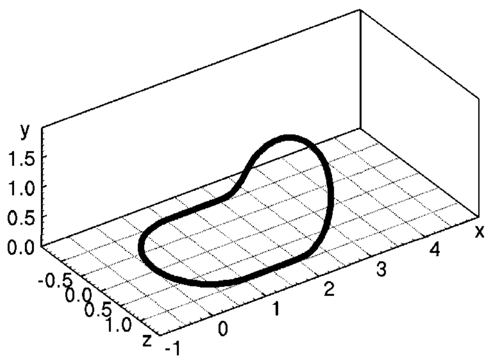

Fig.1 Representation of hairpin vortex by a vortex filament

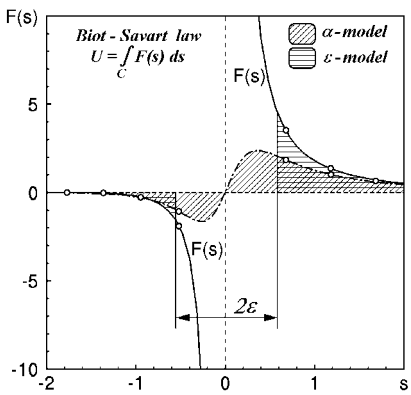

Fig.2 Integration according to the Biot-Savart law

Advantages of the model
- Numerical integration by the formulas of the various order,
- It accounts the cross-section size of the vortex tube,
- Representation of the vortex tube by a smooth curve,
- Description of vortex tube by small number of markers.

Disadvantages of the model
- Increasing the order of integration methods leads to longer calculation.

RESULTS

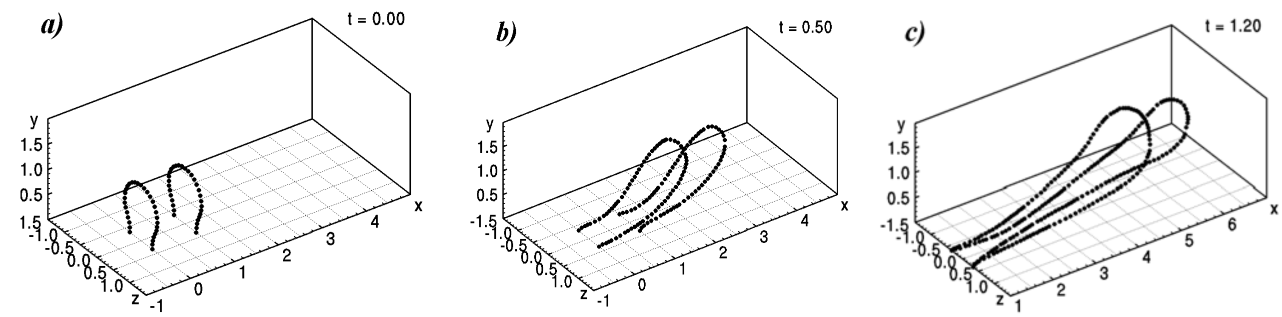

Fig.3 Dynamics of two hairpin vortices above a rigid wall in a shear flow

CONCLUSION

We addressed the global vorticity dynamics by representing each hairpin vortex as a filament with a `core parameter', interacting via the Biot-Savart law. The contour kinematic spline method for tracing the vortex filaments in a shear flow over a rigid wall was developed. Special attention is paid to the soliton-like behaviour of the vortex filaments over the rigid plane. Comparisons with experimental results and DNS data show a good correspondence. Although an extreme idealization, the analytical model of vortex filaments appears to shed considerable light on what to expect in the laboratory experiments. The results obtained for the concentrated hairpin vortex structures confirm von Kármán [5] words that “many peculiarities of real flow can be understood based on the notion of existence of separated vortices in the flow and the laws of motion of such vortices in an ideal fluid.”

References

[1] Robinson S. K.: Coherent motion in the turbulent boundary layer. *Annu. Rev. Fluid Mech* **23**:601-639, 1991.

[2] Adrian R. J., Meinhart C.D., Tomkins C.D.: Vortex organization in the outer region of the turbulent boundary layer. *J. Fluid Mech* **422**:1-54, 2000.

[3] Perry A. E., Chong M.S.: On the mechanism of wall turbulence. *J. Fluid Mech* **119**:173-217, 1982.

[4] Spalart P. R.: Direct simulation of a turbulent boundary layer up to $Re_\theta = 1410$. *J. Fluid Mech* **187**:61-98, 1988.

[5] Kármán Th. von: Mechanische Ähnlichkeit und Turbulenz. *Nachr. Ges. Wissen. Göttingen. Math-Phys.Kl*:17-25, 1930.