FM25L_11615

XXI International Congress of Theoretical and Applied Mechanics
Warsaw, Poland, August 15–21, 2004

Curvature Instability of a Vortex Ring

Yasuhide Fukumoto(1), Yuji Hattori(2)

(1) *Graduate School of Mathematics, Kyushu University, Japan*
(2) *Faculty of Engineering, Kyushu Institute of Technology, Japan*

A new instability mechanism is found for Kelvin's vortex ring, which may surpass the Widnall instability for thin cores. The basic state is a steady asymptotic solution of the Euler equations, in powers of the ratio ε of core radius to ring radius, for an axisymmetric vortex ring with vorticity proportional to the distance from the symmetric axis. In this, the effect of ring curvature of $O(\varepsilon)$ precedes the $O(\varepsilon^2)$ straining field. We show that the $O(\varepsilon)$ field causes a parametric resonance instability between a pair of Kelvin waves whose azimuthal wavenumbers are separated by one. The eigenvalue problem of the Euler equations is solved explicitly, in terms of the Bessel and the modified Bessel functions, and the maximum growth rate is found to be $165/256\,\varepsilon$. This result is reinforced by invoking the geometric optics approach for local stability. The closed-form solution facilitates nonlinear stability analysis.

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