

## Nonlinear Scale-Free Cascades and Their Stationary Spectra

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A wide variety of complex multi-scale mechanical phenomena produce power-law distributions: the turbulence spectra of Kolmogorov, Kraichnan, and Batchelor, and turbulence intermittency spectra; size distributions of coagulating particles in multiphase flows, coalescing drops in clouds, and cracks in solids; the asteroid size distributions due to collisional fragmentation – to mention just a few examples. We propose that all such processes can be viewed from the vantage point of nonlinear scale-free conservative cascades, which we define in a novel rigorous framework and classify by the values of three indices. Next, we show how stationary power-law spectra arise in our framework, and derive an expression for the power-law exponent,  $\tau$ . For many cascades  $\tau$  is found to depend only on the indices values by a simple algebraic formula. This formula unifies complex multi-scale phenomena, and various turbulence spectra in particular, that had been seen as different before.

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