EMPIRICAL GALERKIN MODELS FOR INCOMPRESSIBLE FLOW — PRESSURE-TERM AND 'SUBGRID' TURBULENCE REPRESENTATIONS

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Summary Necessary ingredients of *accurate* empirical Galerkin models for incompressible free and wall-bounded shear flows are discussed. These models are based on the Karhunen-Loève (K-L) decomposition of a Navier-Stokes simulation and a Galerkin projection on the Navier-Stokes equation. Specifically, a novel analytical pressure-term representation is first developed and shown to be necessary for accurate Galerkin systems of near-field wakes and of mixing layers. Secondly, a hierarchy of 'subgrid' turbulence models based on Rempfer's (1991) modal eddy viscosities is presented and shown to be helpful if a low-dimensional K-L ansatz does not resolve a significant portion of the fluctuation energy. Finally, the role of 'missing' phase space directions in the K-L ansatz is revisited and additional modes are proposed. The proposed generalizations and improvements have been integrated in a modular Galerkin 'tool-box' with a hierarchy of procedures to determine model coefficients.

INTRODUCTION

Most 'empirical' Galerkin models are based on Karhunen-Loève (K-L) decompositions of numerical simulations or experimental data [1]. Low-dimensional Galerkin models of coherent structures are often helpful for testing physical understanding. More recently, many low-dimensional modelling efforts are also targeting flow control applications for two main reasons: Such 'plant models' allow the use of all the powerful tools of control theory and their simplicity allows quick exploratory actuation studies.

FRAME-WORK OF THE EMPIRICAL GALERKIN METHOD

The frame-work of the empirical Galerkin method [1] is generalized by adding physical modes to the K-L decomposition and by incorporating additional physical processes in the Galerkin system. The generalized Galerkin approximation for the velocity field of an incompressible flow is taken to be

$$\mathbf{u} = \sum_{i=0}^{N_{KL}} a_i \, \mathbf{u}_i + \sum_{i=N_{KL}+1}^{N} a_i \, \mathbf{u}_i \tag{1}$$

where $a_0 = 1$ and $i = 0, \ldots, N_{KL}$ are indices of the K-L modes $\mathbf{u}_i$ and their Fourier coefficients a_i. Non-empirical modes may be added and are indicated by the index $i = N_{KL}+1, \ldots, N$. The Galerkin system is given by

$$\frac{d}{dt} a_i = \sum_{j=0}^{N} (\nu \, l_{ij} + l_{ij}^{+}) \, a_j + \sum_{j=0}^{N} \sum_{k=0}^{N} (q_{ijk} + q_{ijk}^{+}) \, a_j \, a_k \tag{2}$$

where $\nu = 1/Re$. The coefficients l_{ij} arise from the viscous term, and q_{ijk} from the convection term. Coefficients with the superscript '$+$' may be added to incorporate the effect of the pressure term and to represent non-resolved 'subgrid' fluctuations. The 'standard' model is defined by $N = N_{KL}$, $l_{ij}^{+} \equiv 0$, and $q_{ijk}^{+} \equiv 0$.

RESULTS FOR FREE AND WALL-BOUNDED SHEAR FLOWS

Pressure-term representation

A pressure-term representation has been analytically derived from the pressure Poisson equation [2][3]. The main ingredients are the correct implementation of the boundary conditions and a computationally manageable algorithm. This pressure model leads to an additional quadratic term q_{ijk}^{+} in Eq. (2). This improvement can have a drastic effect on the accuracy of the Galerkin model for convectively unstable shear flows (see Fig. 1) — even if the fluctuation energy is fully resolved by the standard Galerkin approximation with $N = N_{KL}$. On the other hand, incorporating the pressure term in models of self-excited flows, such as near-wakes, appears less important.

Unresolved 'subgrid' turbulence representation

At higher Reynolds-numbers, a low-dimensional ansatz (1) can only resolve a fraction of the fluctuation energy. The effect of the neglected 'subgrid' fluctuations on the resolved coherent structures is modelled by an ansatz of the form

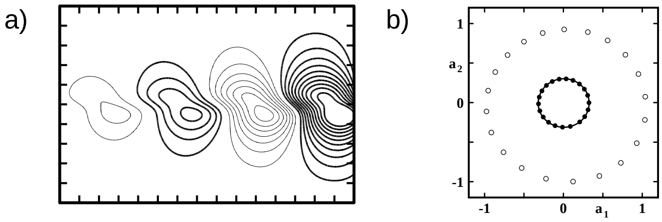

Figure 1. 4-dimensional Galerkin model of a 2D Kelvin-Helmholtz instability with the inlet profile $u = 2/3 + 1/3 \tanh y$. (a) Streamlines of the fluctuation in the computational domain. The tick marks are separated by the vorticity thickness. (b) Phase portraits of the Navier-Stokes simulation and Galerkin models based on the first two Fourier coefficients a_1, a_2 of the Galerkin approximation (1). They show that the limit cycle of the Galerkin model with pressure-term representation ('•') essentially coincides with the Navier-Stokes attractor (solid curve), whereas the omission of the pressure term ('○') gives rise to amplitudes which are far too large.

$l^+_{ij} = \nu_{T,i} l_{ij}$ [4]. This ansatz has been generalized by a hierarchy of algorithms for the determination of the modal eddy viscosities $\nu_{T,i}$ directly from simulation data and without solution matching [5]. Fig. 2 shows the improvements achieved for a transitional wall-bounded shear-layer — including also a pressure-term representation.

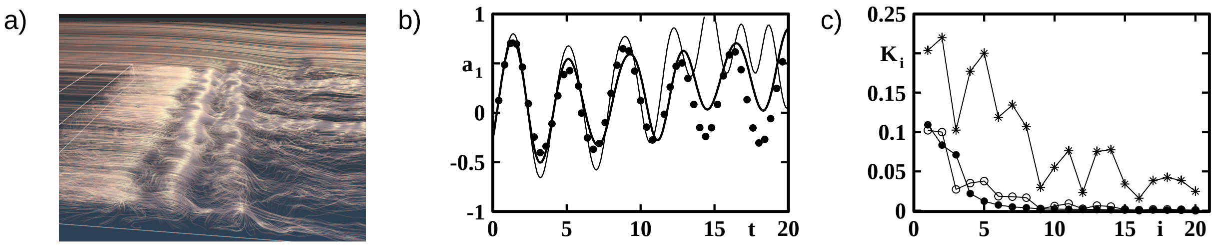

Figure 2. 20-dimensional Galerkin model of the transitional flow over a back-ward facing step at $Re_h = 3000$. (a) Snapshot of the streamlines (the step is indicated by a white frame). (b) the Galerkin model with the turbulence representation (thick curve) follows the LES simulation (•) over on a longer time than the model without turbulence representation (thin curve). (c) Modal energy distribution K_i (i being the mode index). The LES results (•) are significantly better predicted with the enhanced turbulence model (thick curve) than without (thin curve).

Re-construction of missing phase-space directions

Finally, the range of validity of the Galerkin model is enhanced to not only capture the post-transient perturbation dynamics but also the basic (unstable) steady Navier-Stokes solution and the transient dynamics. This is achieved by adding non-empirical modes obtained from a weakly non-linear stability analysis [6].

CONCLUSIONS

The proposed generalisations and improvements of the empirical Galerkin method include (i) a pressure-term representation for open fbws, (ii) a representation of unresolved 'subgrid' turbulence at high-Reynolds numbers and (iii) the addition of non-empirical modes for non-equilibrium conditions. These improvements are additive and therefore it has been possible to incorporate them in a modular 'tool-box' with different levels of simplifi cations. This tool-box signifi - cantly enlarges the class of fbws for which Galerkin models can be usefully constructed. In addition, the dynamic range of the Galerkin system has been markedly enhanced by the proposed improvements. This effect is exploited in fbw control applications [7] which shall be presented in another contribution at this conference.

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