EXPERIMENTAL STUDY OF ROTOR-STATOR FLOWS WITH CENTRIPETAL FLUXES

Sébastien Poncet, Marie-Pierre Chauve, Patrice Le Gal
IRPHE UMR 6594-CNRS / Université d'Aix-Marseille I et II, Technopôle Château Gombert, 49 rue F. Joliot Curie B.P.146, 13384 Marseille cédex 13, France

Summary The evolution of the entrainment coefficient K of the rotating fluid in a rotor-stator cavity, is studied with an imposed centripetal flux and according to the flow control parameters. Measurements are realised by a two component laser Doppler anemometry (L.D.A.). It is shown that the coefficient K depends on a local flow rate coefficient Cq_r of the fluid according to a $5/7$ power law whose coefficients depend on the value of the velocity at the entry of the cavity. A theoretical analysis confirms the asymptotic behavior.

INTRODUCTION

Rotating flows are studied since more than one century [2] because they are present in many rotating machines. Of prime importance for these flows is the rate of rotation K (or entrainment coefficient) of the fluid in the rotor-stator cavity. The values taken by this coefficient determine in fact the rotating machine operating conditions. Following the analysis performed by R. Gassiat in his thesis [3], our study relates to the determination of K when a centripetal flux is added on the rotating flow. We show that a local flow rate coefficient Cq_r is the similarity parameter of the flow and can be used directly to calculate K. As these values are connected to the pressure gradient across the cavity, the knowledge of K is particulary interesting from an industrial point of view.

EXPERIMENTAL SET-UP

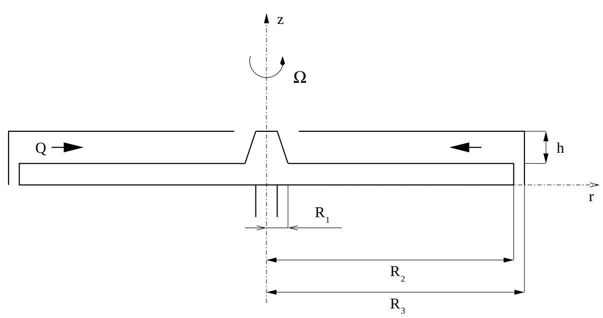

Figure 1. Annular cavity with $\Omega = 0 - 63\ rad.s^{-1}$, $Q = 0 - 4\ l.s^{-1}$, $h = 0 - 12\ mm$, $R_1 = 37.5\ mm$, $R_2 = 250\ mm$, $R_3 = 253\ mm$

The cavity consists of a fixed disk (stator) and of a rotating disk (rotor) which rotates at the uniform angular velocity Ω. The height of the cavity h is variable. The flow depends mainly on three control parameters which are : the Reynolds number $Re = \Omega r^2/\nu$ (r the local radius, ν the water kinematic viscosity), the Rossby number $Ro = Q/(2\pi\rho\Omega R_2^2 h)$ (ρ the water density) and the aspect ratio $G = h/R_2$. A centripetal and variable flux Q is imposed on the basic flow. At the cavity entry, the fluid rotation is ensured by a complementary device linked to the rotor. At rest, the cavity is maintained with a pressure of 2 bars to avoid cavitation effects. The temperature is also maintained constant by a special water cooling device. The flow is supposed to be axisymmetric. By L.D.A., the two velocities components, radial velocity V_r and tangential velocity V_θ are measured in the frame (r, z). For a given radius the rate of rotation K of the fluid is defined by the ratio of the tangential velocity of the fluid in the rotating core and the disk velocity at the same radius position ($K = V_\theta(r)/(\Omega r)$).

RESULTS

The challenge is to find a relation giving the rate of rotation of the fluid K versus h, Ω and Q by using a new coefficient called the local flow rate coefficient whose expression will be determined in the following.

Definition of the local flow rate coefficient Cq_r

It is recalled that we consider a flow with two boundary layers separated by a central rotating core : the Ekman layer on the rotor and the Bödewadt layer on the stator. First we seek an expression for the thickness of the Ekman layer [5]. We consider a volume element of δ_E in height and dS in surface inside the boundary layer of the rotating disk. In this volume element the balance between the centrifugal force and the shearing stress is expressed by :

$$\tau_0 \, \sin\alpha \, dS = \rho \, r \, \Omega^2 \, \delta_E \, dS \tag{1}$$

where α is the angle formed by the shearing stress at the wall τ_0 and the tangential direction.
If it is supposed that the velocity profile evolves according to a $1/7$ power law, the tangential component of shearing stress is given by (The external velocity of the boundary layer is the core velocity decreased by the disk velocity) :

$$\tau_0 \, \cos\alpha = \rho \, ((K-1) r \, \Omega)^{7/4} \, (\nu/\delta_E)^{1/4} \tag{2}$$

By supposing that the angle α of stream lines inclination remains constant along the radius r, we find the expression of the Ekman layer thickness : $\delta_E \sim r \, (K-1)^{7/5} \, (\Omega r^2/\nu)^{-1/5}$.

Secondly we seek an expression for the Bödewadt layer thickness. Contrary to [1], it is supposed that the radial friction is controlled in this boundary layer by the radial flux :

$$\tau_0 \, \sin\alpha = \rho \, (Q/(2\pi r \delta_{Bo}))^{7/4} \, (\nu/\delta_{Bo})^{1/4} \tag{3}$$
$$\tau_0 \, \cos\alpha = \rho \, (K \, r \, \Omega)^{7/4} \, (\nu/\delta_{Bo})^{1/4} \tag{4}$$

The combination of these two last relations gives the expression of the Bödewadt layer thickness $\delta_{Bo} \sim Q/(2\pi r^2 \Omega K)$. Now by considering that radial velocity in the rotating core is zero (that is experimentally verified), the continuity equation reads:

$$\overline{V_{Bo}} \, \delta_{Bo} + \overline{V_E} \, \delta_E = Q/(2\pi r) \tag{5}$$

$\overline{V_{Bo}}$ and $\overline{V_E}$ are mean velocities proportional to the maximum values of the boundary layers velocities (for large K) :

$$\overline{V_{Bo}} = \gamma Q/(2\pi r \delta_{Bo}) = \gamma \, K \, \Omega \, r \qquad \overline{V_E} = \beta r \Omega \tag{6}$$

By using (5) and (6), we get: $K \sim (\frac{Q}{2\pi r^3 \Omega}(\frac{\Omega r^2}{\nu})^{1/5})^{5/7}$, which gives finally the required expression of the similarity parameter Cq_r :

$$Cq_r = \frac{Q}{2\pi r^3 \Omega} (\Omega r^2/\nu)^{1/5} \tag{7}$$

Under these conditions we can write $K = (a \times Cq_r + b)^{5/7}$, where a and b are constants determined experimentaly. It can be noticed that for $r = R_2$, we find the same expression for Cq_r that in [4].

Experimental results

The experimental device previously described have been used to test relation (7) by determining K for many values of Q, Ω and r and for two values of the gap h. The whole of the experimental results are in good agreement (see fig.2) with the $5/7$ power law with $a = 11.1$ and $b = 0.31$. For our experiment, this law is validated in the range of Reynolds number $[6.92 \times 10^5 - 4.15 \times 10^6]$ and of Rossby number $[2 \times 10^{-6} - 6.6 \times 10^{-5}]$.

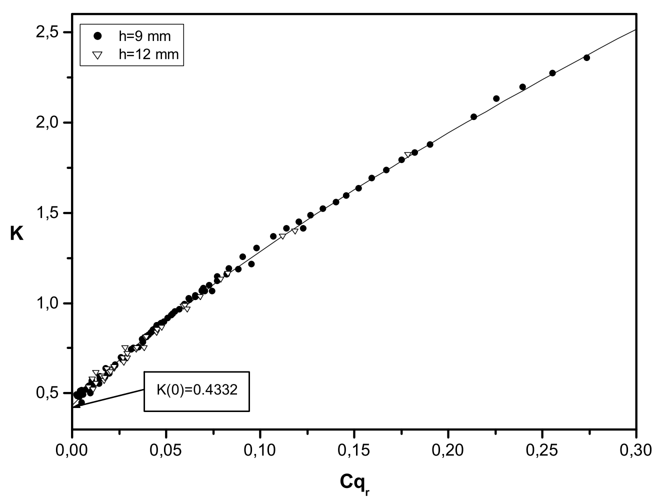

Figure 2. $5/7$ power law giving K versus Cq_r.

References

[1] Daily J.W., Nece R.E. : Chamber dimension effects on induced flow and frictional resistance of enclosed rotating disks. *A.S.M.E. J. Basic Eng.* **82**:217–232, 1960
[2] Ekman V.W.: On the influence of the Earth's rotation on ocean currents. *Arkiv. Mat. Astr. Fys.* **2**(11):1–52, 1905.
[3] Gassiat, M.R.: Etude expérimentale d'écoulements centripètes avec prérotation d'un fluide confiné entre un disque tournant et un carter fixe. *PhD Thesis*, Univ. Aix-Marseille II, Marseille, 2000.
[4] Kurokawa J., Toyokura T. : Axial thrust, discfriction torque and leakage loss of radial flow turbomachinery. *Int. Conference on pump and turbine design and development*, Glasgow, 1976
[5] Schlichting H.: Boundary-Layer Theory. McGraw-Hill, NY 1979.