STOCHASTIC MODEL OF THE CONDITIONAL LAGRANGIAN ACCELERATION OF A FLUID PARTICLE IN DEVELOPED TURBULENCE

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Summary The random intensity of noise approach to 1D Laval-Dubrulle-Nazarenko model is used to describe Lagrangian acceleration of a fluid particle in developed turbulence. Intensities of noises entering nonlinear Langevin equation are assumed to depend on random velocity fluctuations in an exponential way. The conditional acceleration PDF, variance, and mean are found to be in a good qualitative agreement with the recent high-Re Lagrangian experimental data.

Phenomenological approaches [1, 2] were used [3] to describe Lagrangian acceleration of a fluid particle in developed turbulent flow within the framework of Langevin type equation; see also [4]. Recent one-dimensional (1D) stochastic particle models and some refinements [5, 6] were reviewed in [7]. Some toy models of developed turbulence suffer from the lack of physical interpretation deduced from turbulence dynamics [8].

Recently [7, 9] we have shown that the 1D Laval-Dubrulle-Nazarenko (LDN) type toy model [10, 11] of the acceleration evolution with the model turbulent viscosity $\nu_{\rm t}$ and coupled delta-correlated Gaussian multiplicative and additive noises is in a good agreement with the recent high-precision Lagrangian experimental data on acceleration statistics; $R_\lambda = 690$, the normalized acceleration range is $[-60, 60]$, Kolmogorov scale is resolved [12–14]. The longstanding Heisenberg-Yaglom scaling, $\langle a^2\rangle = a_0\bar{u}^{9/2}\nu^{-1/2}L^{-3/2}$, was confirmed experimentally [12] to a very high accuracy, for about seven orders of magnitude in the acceleration variance, or two orders of the *rms* velocity $\bar{u}$, at $R_\lambda > 500$. Long-time correlations and the occurrence of very large fluctuations at small scales dominate the motion of a fluid particle, and this leads to a new dynamical picture of turbulence [15, 16]. We focus on modeling the acceleration statistics conditional on velocity fluctuations u presented recently in [14].

The original 3D and 1D LDN models were formulated both in the Lagrangian and Eulerian frameworks for small-scale velocity increments in time and space respectively. They are based on a stochastic kind of Batchelor-Proudman rapid distortion theory approach to the 3D Navier-Stokes equation [10], and thus have a deductive support from turbulence dynamics. The random intensity of noise (RIN) approach [7, 9] provides an extension of the above 1D model in the limit of small time scale τ for which Lagrangian velocity increments are proportional to τ: $u(t+\tau) - u(t) = \tau a(t)$. The main idea of RIN approach is simply to account for the recently established two well separated Lagrangian autocorrelation time scales for the velocity increments [15] and assume that certain model parameters, such as intensities of the noises, fluctuate at the *long* time scale due to Lagrangian velocity u. A simple 1D model can shed some light to properties of 3D LDN model of Lagrangian dynamics.

We give only a brief sketch of the 1D LDN model and refer the reader to [7, 10] for more details; see also [17]. This toy model can also be viewed as a passive scalar in a compressible 1D flow [10]. We use probability density function (PDF) obtained as a stationary solution of the Fokker-Planck equation associated to the Langevin equation for the component of Lagrangian acceleration $a(t)$: $\partial a/\partial t = (\xi - \nu_{\rm t}k^2)a + \sigma_\perp$ [10]. In 3D LDN model, $\xi(t)$ is related to the velocity derivative tensor and $\sigma_\perp(t)$ describes a forcing of small scales by large scales via the energy cascade mechanism. In 1D LDN model, these are approximated by external Gaussian white noises,

$$\langle\xi(t)\rangle = 0,\ \langle\xi(t)\xi(t')\rangle = 2D\delta(t-t'),\ \langle\sigma_\perp(t)\rangle = 0,\ \langle\sigma_\perp(t)\sigma_\perp(t')\rangle = 2\alpha\delta(t-t'),\ \langle\xi(t)\sigma_\perp(t')\rangle = 2\lambda\delta(t-t'), \quad (1)$$

that is partially justified by DNS [10]. The acceleration PDF can be calculated exactly [7], with the result

$$P(a) = C\exp[-\nu_{\rm t}k^2/D + F(c) + F(-c)](Da^2 - 2\lambda a + \alpha)^{1/2}(2Bka + \nu_{\rm t}k^2)^{-2B\lambda k/D^2}, \quad (2)$$

for constant parameters. Here, we have denoted $\nu_{\rm t} = \sqrt{\nu_0^2 + B^2a^2/k^2}$, C is normalization constant,

$$F(c) = \frac{c_1k^2}{2c_2D^2c}\ln\left[\frac{2D^3}{c_1c_2(c - Da + \lambda)}(B^2(\lambda^2 + c\lambda - D\alpha)a + c(D\nu_{\rm t}^2k^2 + c_2\nu_{\rm t}))\right],\ c = -i\sqrt{D\alpha - \lambda^2}, \quad (3)$$

$$c_1 = B^2(4\lambda^3 + 4c\lambda^2 - 3D\alpha\lambda - cD\alpha) + D^2(c+\lambda)\nu_0^2k^2,\ c_2 = \sqrt{B^2(2\lambda^2 + 2c\lambda - D\alpha)k^2 + D^2\nu_0^2k^4}. \quad (4)$$

Without loss of generality one can put, in a numerical study, $k = 1$ and the additive noise intensity $\alpha = 1$ by rescaling of the multiplicative noise intensity $D > 0$, the turbulent viscosity parameter $B > 0$, the kinematic viscosity $\nu_0 > 0$, and the cross correlation λ [7], and make a fit of $P(a)$ to the experimental data. The particular cases $B = 0$ and $\nu_0 = 0$ at $\lambda = 0$ were studied in detail in [7]. Nonzero λ is responsible for an asymmetry of the PDF (2) and in 3D picture corresponds to a correlation between stretching and vorticity (the energy cascade); in the Eulerian framework, $\langle(\delta u)^3\rangle$ is proportional to λ, in accord to a kind of Karman-Howarth equation [10]. Since the experimental unconditional and conditional distributions, $P_{\rm exp}(a)$ and $P_{\rm exp}(a|u)$ at $u = 0$, were found to be approximately of the same stretched exponential form revealing strong Lagrangian intermittency [14] we use the result of our fit [9] of the PDF (2) to $P_{\rm exp}(a)$ [13] measured with 3% relative uncertainty for $|a| \leq 10$. This implies the following rescaled parameter set: $D = 1.100$, $B = 0.155$, $\nu_0 = 2.910$, $\lambda = -0.005$ ($k = 1$,

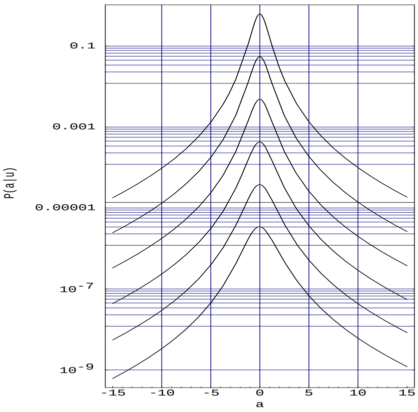

FIG. 1: The conditional acceleration PDF $P(a|\alpha(u), \lambda(u))$ given by Eq. (2). $u = 0$ (top curve), 0.25, 0.50, 0.75, 1.00, 1.19 (bottom curve).

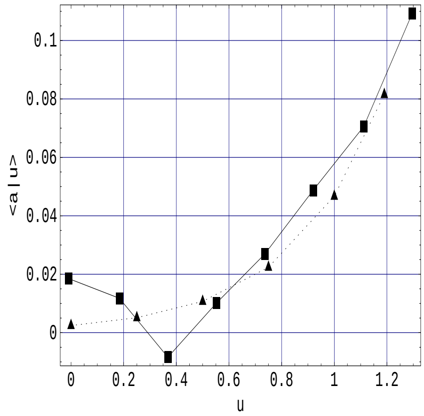

FIG. 2: The conditional mean acceleration vs u. Triangles: $\langle a|u\rangle/\langle a^2|0\rangle^{1/2}$ for $\alpha = e^{|u|}$ and $\lambda = -0.005e^{3|u|}$. Boxes: the experimental data on $\langle a|u\rangle/\langle a^2\rangle^{1/2}$ [14].

$\alpha = 1$, $C = 3.230$). A fit to the *conditional* distribution $P_{\rm exp}(a|0)$ would yield a different set of values of the parameters. The used fit is however justified on a qualitative level. We assume that the parameters α and λ entering (2) depend on the amplitude of (normalized) Lagrangian velocity fluctuations u, while D, B, and ν_0 are taken to be fixed at the fitted values ($k = 1$). An exponential form of $\alpha(u)$ has been proposed in [7] and was found to be relevant from both the (K62) phenomenological and experimental points of view. Particularly, such a form leads to log-normal RIN model when u is independent Gaussian distributed with zero mean (PDF $g(u)$), and yields the acceleration PDF whose low-probability tails are in agreement with experiments [3, 7]; the marginal PDF is $P_{\rm m}(a) = \int_{-\infty}^{\infty} P(a|u)g(u)du$. Remarkably, this form is found to provide appreciable increase of the *conditional acceleration variance* $\langle a^2|u\rangle$ with increasing $|u|$ [7] that meets the experimental data [14]. Guided by these observations the simplest choice is to try an exponential dependence for $\lambda(u)$. Particularly, in the present description we use $\lambda(u) = -0.005e^{3|u|}$ and $\alpha(u) = e^{|u|}$. This allows us to deal with the acceleration statistics which exhibits a strongly non-Gaussian character in both the unconditional and conditional cases.
The conditional PDF is given by (2) treated in the form $P(a|\alpha(u), \lambda(u))$. The constant C in (2) is calculated for each value of u. For the normalized velocity fluctuations values $u = 0$, 0.25, 0.50, 0.75, 1.00, 1.19 the distributions $P(a|\alpha(u), \lambda(u))$ and mean accelerations $\langle a|u\rangle/\langle a^2|0\rangle^{1/2}$ are shown in Figs. 1 and 2. One observes a good qualitative correspondence of the obtained *conditional PDFs* (Fig. 1, shifted for clarity) with the experimental curves (Fig. 6a in [14]). Both the variance and skewness of $P(a|u)$ increase for bigger $|u|$. While the increase of the variance (related to the increase of $\alpha(u)$) is readily seen, the increase of the skewness (related to the increase of $\lambda(u)$) results in a rather small change of the shape of distribution despite the fact that λ varies by about two order of magnitude (from $|\lambda|$=0.005 to 0.18). Such a change can be however readily seen in a plot of the contribution to fourth order moment, $a^4P(a)$ (see Fig. 4 in [9]). The obtained *conditional mean acceleration* $\langle a|u\rangle/\langle a^2|0\rangle^{1/2}$ plotted in Fig. 2 is also in a good qualitative agreement with the experimental dependence $\langle a|u\rangle/\langle a^2\rangle^{1/2}$ (Fig. 6b in [14]). The mean acceleration is zero for a symmetrical distribution and for homogeneous isotropic turbulence. One observes a rather small relative increase of the mean acceleration for bigger $|u|$ that eventually reflects a coupling of the acceleration to large scales of the studied flow [10, 16]. We note that the experimental $\langle a|u\rangle/\langle a^2\rangle^{1/2}$ exhibits small asymmetry with respect to $u \to -u$. In summary, the presented 1D model is shown to capture main features of the observed conditional acceleration statistics.

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