

Helicity Dynamics of Vortex Filaments

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We study the singular filament solutions of the fluid equation obtained by requiring the fluid momentum to also be its vorticity. The Lagrangian in Hamilton's principle for this equation is the helicity: the linkage of the vorticity field. This fluid equation admits a Hamiltonian formulation that preserves the linkage of the curl of vorticity and has singular filament solutions on space curves $\mathbf{x} = \mathbf{R}(t, s)$ whose motion is described in the local induction approximation by a modified da Rios-Betchov equation $\dot{\mathbf{R}}(t, s) = (-c/4\pi)\hat{P}\mathbf{R}_{,sss}$, where the projection \hat{P} causes the motion to be transverse to the filament. Thus, we determine the Hamiltonian dynamics in the Lagrangian fluid description describing a massless filament of vorticity $\omega = \text{curl}\mathbf{u}$ supported along a space curve $\mathbf{x} = \mathbf{R}(\mathbf{a}, t)$ that moves without slipping in the incompressible flow induced by its own helicity.

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